Karlsruhe Institute of Technology

Seminar of the Work Group
Nonlinear Partial Differential Equations
SS 24

May 15th, 2024, 11:30 - 12:15
Seminar room: SR 3.069

Evolving Heterogeneous Elastic Wires

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Abstract

The elastic energy of a bending-resistant interface depends both on its geometry and its material composition. We consider such a heterogeneous interface in the plane, modeled by a curve equipped with an additional density function. The resulting energy, which depends on material parameters, captures the complex interplay between curvature and density effects and resembles the Canham–Helfrich functional.

We study a family of planar curves with density evolving in time according to the steepest descent associated to this energy. Describing the curves by their inclination angle, the L^2-gradient flow is a nonlocal coupled parabolic system of second order. We shortly discuss local well-posedness via maximal regularity theory on time-dependent little Hölder spaces. Once global existence is established, convergence of solutions follows with a constrained Łojasiewicz–Simon gradient inequality. We show that the (non)preservation of quantities such as convexity and positivity of the density depends delicately on the choice of material parameters. The same applies for the asymptotic behavior of the system.

This talk is based on joint work with Anna Dall'Acqua (Ulm University), Fabian Rupp (University of Vienna) and Gaspard Jankowiak (University of Graz).

References

[1] A. Dall'Acqua, L. Langer, and F. Rupp. A dynamic approach to heterogeneous elastic wires. *J. Differ. Equ.*, **392** (2024), 1–42.

[2] A. Dall'Acqua, G. Jankowiak, L. Langer, and F. Rupp. Conservation, convergence, and computation for evolving heterogeneous elastic wires. arXiv preprint, accepted for publication in *SIAM J. Math. Anal.*