Non-selfadjoint Spectral Problems Related to Self-similar Blowup in Nonlinear Wave Equations

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Abstract

We consider the wave equation with a power nonlinearity
\[ u_{tt} - \Delta u = u^p \]
with initial profiles \( u(x, 0) \) and \( u_t(x, 0) \), \( x \in \mathbb{R}^3 \), \( t \geq 0 \), and \( p > 1 \) an odd integer. In order to investigate the blowup dynamics we look for radial self-similar blowup solutions of the form
\[ u(x, t) = (T - t)^{-\frac{2}{p-1}} U \left( \frac{|x|}{T-t} \right), \quad T > 0, \]
with a smooth, radial profile \( U \). In particular, we are interested in stability properties of such solutions. This gives rise to analyzing the spectrum of the linearized operator, i.e. to the eigenvalue problem:
\[ \mathcal{L}u = \lambda u, \]
where \( D(\mathcal{L}) \subset H^2_{\text{rad}}(B^3) \times H^1_{\text{rad}}(B^3), H^k_{\text{rad}}(B^3) := \{ u \in H^k(B^3) : u \text{ is radial} \} \),
\[ \mathcal{L} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} := \begin{pmatrix} -\rho u_1' + \alpha u_2 + u_2 \\ u_1' + \frac{2}{\rho} u_2 - (\alpha + 1) u_2 + V(\rho) u_1 \end{pmatrix}, \]
\[ \rho = \frac{|x|}{T-1}, \quad V(\rho) = pU(\rho)^{p-1} \quad \text{and} \quad \alpha = \frac{2}{p-1}. \]
We are interested in excluding eigenvalues of \( \mathcal{L} \) in parts of the right complex half plane, which is ongoing work together with B. Schöckhuber, Y. Watanabe, M. Plum and M.T. Nakao. In this talk, we will show (as a partial result) that all eigenvalues \( \lambda \) in the half-plane \( \{ \text{Re}(z) > 1 - \alpha \} \) are real, which constitutes a substantial advantage for the computation of the desired eigenvalue exclusions. We also provide a global existence region for the complex eigenvalues and upper bounds for the real eigenvalues and show the principle, how we can exclude eigenvalues in such a region.