Karlsruhe Institute of Technology

Seminar of the Work Group
Nonlinear Partial Differential Equations
SS 24

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Seminar room: SR 3.069

Non-selfadjoint Spectral Problems Related to Self-similar Blowup in Nonlinear Wave Equations

Kaori Nagatou, KIT

Abstract

We consider the wave eqaution with a power nonlinearity

$$u_{tt} - \Delta u = u^p$$

with initial profiles $u(x,0)$ and $u_t(x,0)$, $x \in \mathbb{R}^3$, $t \geq 0$, and $p > 1$ an odd integer. In order to investigate the blowup dynamics we look for radial self-similar blowup solutions of the form

$$u(x,t) = (T-t)^{-\frac{2}{p-1}} U\left(\frac{|x|}{T-t}\right), \qquad T > 0,$$

with a smooth, radial profile U. In particular, we are interested in stability properties of such solutions. This gives rise to analyzing the spectrum of the linearized operator, i.e. to the eigenvalue problem:

$$\mathcal{L}\mathbf{u} = \lambda \mathbf{u},$$

where $D(\mathcal{L}) \subset H^2_{\text{rad}}(B^3) \times H^1_{\text{rad}}(B^3)$, $H^k_{\text{rad}}(B^3) := \{\mathbf{u} \in H^k(B^3) : \mathbf{u} \text{ is radial}\}$,

$$\mathcal{L}\begin{pmatrix} u_1 \\ u_2 \end{pmatrix} := \begin{pmatrix} -\rho u_1'(\rho) - \alpha u_1(\rho) + u_2(\rho) \\ u_1''(\rho) + \frac{2}{\rho} u_1'(\rho) - \rho u_2'(\rho) - (\alpha+1) u_2(\rho) + V(\rho) u_1(\rho) \end{pmatrix},$$

$\rho = \frac{|x|}{T-1}$, $V(\rho) = pU(\rho)^{p-1}$ and $\alpha = \frac{2}{p-1}$. We are itnerested in excluding eigenvalues of $\mathcal{L}$ in parts of the right complex half plane, which is ongoing work together with B. Schörkhuber, Y. Watanabe, M. Plum and M.T. Nakao. In this talk, we will show (as a partial result) that all eigenvalues λ in the half-plane $\{\text{Re}(z) > 1 - \alpha\}$ are *real*, which constitutes a substantial advantage for the computation of the desired eigenvalue exclusions. We also provide a global existence region for the complex eigenvalues and upper bounds for the real eigenvalues and show the principle, how we can exclude eigenvalues in such a region.