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Complete integrability of the Benjamin–Ono equation on the multi-soliton manifolds

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Abstract This presentation is dedicated to describing the complete integrability of the Benjamin–Ono (BO) equation on the line when restricted to every N-soliton manifold, denoted by $\mathcal{U}_N$. We construct generalized action–angle coordinates which establish a real analytic symplectomorphism from $\mathcal{U}_N$ onto some open convex subset of $\mathbb{R}^{2N}$ and allow to solve the equation by quadrature for any such initial datum. As a consequence, $\mathcal{U}_N$ is the universal covering of the manifold of N-gap potentials for the BO equation on the torus as described by Gérard–Kappeler [1]. The global well-posedness of the BO equation in $\mathcal{U}_N$ is given by a polynomial characterization and a spectral characterization of the manifold $\mathcal{U}_N$. Besides the spectral analysis of the Lax operator of the BO equation and the shift semigroup acting on some Hardy spaces, the construction of such coordinates also relies on the use of a generating functional, which encodes the entire BO hierarchy. The presentation is based on Sun [2].

Keywords Benjamin–Ono equation, Action–angle coordinates, Lax pair, Hardy space, Multi-solitons, Toeplitz operator, Universal covering manifold

References

[1] Gérard, P., Kappeler, T. *On the integrability of the Benjamin–Ono equation on the torus*, available on arXiv:1905.01849, 2019, to appear in CPAM.

[2] Sun, R. *Complete integrability of the Benjamin–Ono equation on the multi-soliton manifolds*, available on arXiv 2004.10007.

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