

Seminar of the Work Group
Nonlinear Partial Differential Equations
SS 23

April 18th, 2023, 14:00 - 15:30
Seminar room: SR 3.068

The Validity of the Derivative NLS Approximation for Systems with Quadratic Nonlinearities

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Abstract

The Derivative Nonlinear Schrödinger equation can be derived as an envelope equation via multiple scaling perturbation analysis from dispersive wave systems. It occurs when the cubic coefficient for the associated NLS equation vanishes for the spatial wave number of the underlying slowly modulated wave packet. It is the purpose of this paper to prove that the DNLS equation makes correct predictions about the dynamics of the Klein-Gordon model with a quadratic nonlinearity. The proof is based on energy estimates and normal form transformations. Difficulties occur due to total resonances and second order resonances in the cubic terms and further stable/unstable resonances in the quartic terms. The approximation property holds if the latter resonances are stable. In the unstable case we give a proof of the failure of the DNLS approximation when considering spatially periodic boundary conditions.