Seminar of the Work Group
Nonlinear Partial Differential Equations
SS 23

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Free Boundary Problems via Da Prato - Grisvard Theory

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Abstract

A common way to prove global well-posedness of free boundary problems for incompressible viscous fluids is to transform the equations governing the fluid motion to a fixed domain with respect to the time variable. An elegant and physically reasonable way to do this is to introduce Lagrangian coordinates. These coordinates are given by the transformation rule

$$x(t) = \xi + \int_0^t u(\tau, \xi)\, d\tau\,,$$

where $u(\tau, \xi)$ is the velocity vector of the fluid particle at time τ that initially started at position ξ. The variable $x(t)$ is then the so-called Eulerian variable and belongs to the coordinate frame where the domain that is occupied by the fluid moves with time. The variable ξ is the Lagrangian variable that belongs to time fixed variables. In these coordinates the fluid only occupies the domain Ω_0 that is occupied at initial time $t = 0$.

To prove a global existence result for such a problem, it is important to guarantee the invertibility of this coordinate transform globally in time. By virtue of the inverse function theorem, this is the case if

$$\nabla_\xi x(t) = \mathrm{Id} + \int_0^t \nabla_\xi u(\tau, \xi)\, d\tau$$

is invertible. By using a Neumann series argument, this is invertible, if the integral term on the right-hand side is small in $L^\infty(\Omega_0)$. Thus, it is important to have a global control of this L^1-time integral with values in $L^\infty(\Omega_0)$. If the domain is bounded, this can be controlled by decay properties of the corresponding semigroup operators that describe the motion of the linearized fluid equation. On certain unbounded domains, however, these decay properties are not true anymore. While there are technical possibilities to fix these problems if the boundary is compact, these fixes cease to work if the boundary is non-compact.

As a model problem, we consider the case where Ω_0 is the upper half-space. To obtain estimates of the L^1-time integral we establish a homogeneous version of the celebrated theorem of Da Prato and Grisvard (1975) about maximal regularity in real interpolation spaces. In these lectures, we will describe this homogeneous Da Prato-Grisvard theorem in detail and show how it can be applied to solve problems from fluid mechanics involving a free non-compact boundary.

This is a joint work with Raphaël Danchin, Matthias Hieber, and Piotr Mucha.