Karlsruhe Institute of Technology

Seminar of the Work Group
Nonlinear Partial Differential Equations
SS 23

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Density Patch Problem for Compressible Fluids

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Abstract

The motion of a compressible viscous barotropic fluid is described by the Navier-Stokes system. It is a system of hyperbolic-parabolic mixed-type PDEs. In this talk, we will study the so-called density patch problem: *If we are given a density that is initially discontinuous across a $\mathcal{C}^{1+\alpha}$ curve γ and α-Hölder continuous on the two disjoint components delimited by γ, is this structure preserved in time?*

An important quantity in the mathematical analysis of this system is the so-called effective flux, which was discovered in [Hoff and Smoller, 1985]. More precisely, the mathematical properties of this quantity play a crucial role in the study of the propagation of oscillations in compressible fluids [Serre, 1991], in the construction of weak solutions [Lion, 1996], or the propagation of discontinuity surfaces [Hoff, 2002], to cite just a few examples. In the case of density-dependent viscosities, the behavior of the effective flux degenerates, which renders the analysis more subtle.

1 References

[Hoff, 2002] Hoff, D. (2002). Dynamics of singularity surfaces for compressible, viscous flows in two space dimensions. *Communications on Pure and Applied Mathematics: A Journal Issued by the Courant Institute of Mathematical Sciences*, 55(11):1365-1407.

[Hoff and Smoller, 1985] Hoff, D. and Smoller, J. (1985). Solutions in the large for certain nonlinear parabolic systems. In *Annales de l'Institut Henri Poincaré C, Analyse non linéaire*, volume 2, pages 213-235. Elsevier.

[Lions, 1996] Lions, P.-L. (1996). *Mathematical Topics in Fluid Mechanics: Volume 2: Compressible Models*, volume 2. Oxford Lecture Mathematics and.

[Serre, 1991] Serre, D. (1991). Variations de grande amplitude pour la densite d'un fluide visqueux compressible. *Physica D: Nonlinear Phenomena*, 48(1):113-128