Karlsruhe Institute of Technology

Seminar of the Work Group
Nonlinear Partial Differential Equations
WS 23/24

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Long-wave Approximation of the GP-Hierarchy by the KdV-Hierarchy

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Abstract

We consider the Gross-Pitaevskii (GP) equation in one dimension, a cubic nonlinear Schrödinger equation with non-zero boundary data $|q| = 1$ at infinity. It is well-known that Schrödinger equations can be transformed into a hydrodynamic form via the so-called Madelung transform. We consider similarly derived variables (w_+, w_-) in which the equation resembles two coupled Korteweg De-Vries (KdV) equations. Under the KdV-scaling

$$W_\pm(T, X) = \frac{1}{\epsilon^2} w_\pm \left(\frac{\sqrt{2}t}{\epsilon^3}, \frac{x}{\sqrt{2}\epsilon} \right) \qquad \epsilon > 0 \text{ small},$$

it has been shown ([Béthuel et al., 2009, 2010]) that W_+ and W_- are each approximated by a solution to the KdV-equation in a co-moving frame. We further investigate this connection between (GP) and (KdV). Using the completely integrable structure of both equations, we show that the transmission coefficient, and hence also the higher energies, of (GP), are approximated by those of (KdV). This implies a connection between the respective hierarchies.

1 References

[Béthuel et al., 2009] Béthuel, Fabrice and Gravejat, Philippe and Saut, Jean-Claude and Smets, Didier. (2009). On the Korteweg-de Vries long-wave approximation of the Gross-Pitaevskii equation I. *International Mathematics Research Notices*, 2009(14):2700-2748.

[Béthuel et al., 2010] Béthuel, Fabrice and Gravejat, Philippe and Saut, Jean-Claude and Smets, Didier. (2010). On the Korteweg-de Vries long-wave approximation of the Gross-Pitaevskii equation II. *Comm. Partial Differential Equations*, 35(1):113-164.