Boussinesq Equations With Variable Viscosity Coefficient

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Abstract
The starting point of our talk is the density patch problem for the two-dimensional incompressible inhomogeneous Navier-Stokes equations with variable viscosity coefficient: It is well-known that if the initial density of the fluid is of the form $\rho_0 = 1_{D_0}$ for some bounded domain $D_0$ in $\mathbb{R}^2$, then there exists a weak solution of the Navier-Stokes equation such that for all times, $\rho(t) = 1_{D(t)}$ for some bounded domain $D(t)$ with $|D(t)| = |D_0|$. The question of the density patch problem is whether the regularity of the boundary of the initial domain is preserved throughout time as well. This leads to the question of well-posedness of the Navier-Stokes equations with discontinuous viscosity coefficient. Previous results on this matter pose a smallness condition on the size of the jump of the viscosity. Our goal is to remove this smallness condition. To do so, we consider the Boussinesq approximation to the Navier-Stokes equation and investigate the question of well-posedness for this simplified model.