Seminar of the Work Group
Nonlinear Partial Differential Equations
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WS 24/25

Speaker: Emile Bukieda
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Nonlinear Stability at the Eckhaus Boundary in case of a Shortwave Destabilization

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Abstract

We consider the complex Ginzburg-Landau equation in two spatial dimensions, which depends on two real parameters a and b and admits a family of periodic traveling stripe solutions parametrized by the wave number. We are interested in the nonlinear stability of these solutions against localized perturbations. It is known that, depending on a and b, these waves are either always unstable or there exists a critical value for the wave number, the so-called Eckhaus boundary, above which they are stable and below which they are unstable. If the wave number passes through the Eckhaus boundary from above a short- or longwave destabilization occurs. It has been shown in case of longwave destabilization that periodic traveling waves are still nonlinearly stable at the Eckhaus boundary. Here, we settle the other case and establish nonlinear diffusive stability at the Eckhaus boundary in case of a shortwave destabilization. Our proof relies on iterative L^1-L^∞ estimates on the Duhamel formulation of the perturbation. To his end, we decompose the underlying semigroup which allows to separately handle the interactions of different critical modes in the nonlinear terms. A challenge is that standard L^1-L^∞ estimates are insufficient to control the most critical interaction terms. Inspired by the space-time resonance method, developed for dispersive systems, we uncover a generic absence of space resonances to exploit oscillations in frequency yielding additional decay which allows to close a nonlinear argument.