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SS 2020
20.05.2020

Boundary and Eigenvalue Problems

Exercise sheet 4

Exercise 10

Let $\Omega = (0,1) \times (0,1) \subset \mathbb{R}^2$ and consider the boundary value problem

$$\begin{cases} u_{xy} = 0 & \text{in } \Omega \\ u = u_0 & \text{on } \partial\Omega \end{cases} \tag{1}$$

for $u \in C^2(\Omega) \cap C(\overline{\Omega})$ and given $u_0 \in C(\partial\Omega)$. Prove that the relation

$$u(0,0) + u(1,1) = u(1,0) + u(0,1)$$

holds for every solution of (1) and conclude that there exists a $u_0 \in C(\partial\Omega)$ such that (1) has no solution. Is the operator $L[u] = u_{xy}$ elliptic?

Exercise 11

Let $\Omega \subset \mathbb{R}^n$ be a C^1-domain. Let $A \in C^1(\overline{\Omega}; \mathbb{R}^{n\times n})$, $b \in C(\Omega, \mathbb{R}^n)$, $c, r \in C(\overline{\Omega})$ and $\gamma \in C(\partial\Omega)$. Show that if $u \in C^2(\overline{\Omega})$ satisfies the weak formulation (S) of the boundary value problem

$$\begin{cases} -\operatorname{div}(A\nabla u) + b \cdot \nabla u + cu = r & \text{in } \Omega \\ u = 0 & \text{on } \Gamma_0 \\ \frac{\partial u}{\partial \mu} + \gamma u = 0 & \text{on } \Gamma_1 \end{cases}$$

and $u = 0$ on Γ_0 then u is also a classical solution.

Exercise 12

Consider the boundary value problem

$$-(au')' = 1 \quad (-1 < x < 1),\ u(-1) = u(1) = 0,$$

where $a(x) = \begin{cases} 1 & \text{, if } \ -1 \le x < 0, \\ \frac{1}{2} & \text{, if } \ \ 0 \le x \le 1. \end{cases}$

Determine the weak formulation of this problem and its solution.