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Bifurcation Theory

Problem Sheet 7

Problem 19 (Bifurcation Formulae in finite dimensions)

As in Problem 17 (b), let $A \in \mathbb{R}^{n\times n}$ be symmetric, and $J \in \mathbb{R}^{n\times n}$. We study

$$Ax = \lambda x + |x|^2 Jx \qquad (x \in \mathbb{R}^n, \lambda \in \mathbb{R}). \tag{1}$$

We have seen that, if λ_0 is a simple eigenvalue of A, a continuously differentiable curve of nontrivial solutions $(\hat{x}(s), \hat{\lambda}(s))_{-\delta<s<\delta}$ of problem (1) bifurcates from $(0, \lambda_0)$.

Show that $\hat{\lambda}'(0) = 0$ and calculate $\hat{\lambda}''(0)$.

Problem 20 (Bifurcation Formulae for Problem 18)

Problem 18 shows that, for $n \geq 2$, the ODE

$$-u'' = \lambda u + u^n \qquad \text{on } \mathbb{R}, \tag{2}$$

admits a continuously differentiable curve $(\hat{u}(s), \hat{\lambda}(s))_{-\delta<s<\delta} \subseteq C^2(\mathbb{R}) \times \mathbb{R}$ of nontrivial 2π-periodic solutions bifurcating from $(0, 0)$.

For $n = 2$ and $n = 3$, sketch the bifurcation diagram near $(0, 0)$ by calculating $\hat{\lambda}'(0)$ and, if the first derivative vanishes, $\hat{\lambda}''(0)$.

Problem 21 (Bending of an elastic rod)

The bending of an elastic rod can be described be the boundary value problem

$$\begin{cases} u'' + \lambda \sin(u) = 0 & \text{in } (0, 2\pi), \\ u'(0) = u'(2\pi) = 0. \end{cases} \tag{3}$$

Find all bifurcation points for problem (3). Sketch the bifurcation diagram near each bifurcation point $(0, \lambda_j)$ with $\lambda_j > 0$ using the bifurcation formulae.

In Problems 20 and 21, you need not prove higher-order Fréchet differentiability.