

Problem Sheet 12
Bifurcation Theory
Winter Semester 2022/23
30.1.2023

Problem 30:

A gradient system is a dynamical system of the form

$$x' = -\nabla_x V(\lambda, x)$$

for a given function $V \in C^1(\mathbb{R} \times \mathbb{R}^n; \mathbb{R})$.

- (a) Prove that gradient systems cannot have non-constant periodic solutions.
- (b) Let $V \in C^2(\mathbb{R} \times \mathbb{R}^n; \mathbb{R})$. Show that Hopf bifurcation cannot occur in gradient systems.
- (c) Consider the dynamical system

$$(1) \quad \begin{cases} x'_1 = 2 - x_1 - \lambda x_2^2, \\ x'_2 = 5 - x_2 - 2\lambda x_1 x_2. \end{cases}$$

Are there non-constant periodic solutions to (1)?

Problem 31:

For $g \in C^2(\mathbb{R}^3; \mathbb{R})$ consider the differential equation

$$(2) \quad -u'' = g(\lambda, u, u').$$

Write (2) as a two-dimensional system

$$(3) \quad y' = f(\lambda, y)$$

in the variable $y = (u, u')$. Under which conditions on g are the assumptions (H1)–(H4) satisfied for some given $\lambda_0 \in \mathbb{R}$, $\beta > 0$?