

Problem Sheet 9
Bifurcation Theory
Winter Semester 2022/23
9.1.2023

Problem 23 (Bending an elastic rod):

The bending of an elastic rod can be described by the boundary value problem

$$(1) \quad \begin{cases} u'' + \lambda \sin(u) = 0 & \text{in } (0, 2\pi), \\ u'(0) = u'(2\pi) = 0. \end{cases}$$

Find all bifurcation points for problem (1). Sketch the bifurcation diagram near each bifurcation point $(0, \lambda_j)$ with $\lambda_j > 0$ using the formulas in Corollary 4.6.

Problem 24 (Bifurcation from ∞):

Consider the nonlinear system

$$\begin{cases} (1 - \lambda)x_1 + \frac{x_2}{x_1^2 + x_2^2} = 0, \\ (1 - 2\lambda)x_2 + \frac{x_1}{x_1^2 + x_2^2} = 0. \end{cases}$$

Show that bifurcation from ∞ occurs for $\lambda_0 = 1$ and $\lambda_0 = \frac{1}{2}$.