

**Boundary and Eigenvalue Problems:
6th problem sheet**

Exercise 1

- a) Let $\Omega \subset \mathbb{R}^n$ be a bounded domain. For which $\alpha \in \mathbb{R}, p \in [1, \infty]$ does the function $f(x) = |x|^\alpha$ belong to $W^{1,p}(\Omega)$?
- b) For which $\alpha \in \mathbb{R}, p \in [1, \infty]$ does the function $g(x) = (1 + |x|^2)^\alpha$ belong to $W^{1,p}(\mathbb{R}^n)$?
Check first that if $h \in C^1(\mathbb{R}^n)$ then the classical first order derivatives of h and the weak first order derivatives of h coincide.

Exercise 2

Determine the first order weak derivatives of the function $u(x, y) = |x| + |y|$ on the unit disc in \mathbb{R}^2 .

Exercise 3

Prove that the function $u(x, y) = \text{sign}(x) + \text{sign}(y)$ does not have first order weak derivatives in \mathbb{R}^2 but the weak derivative $\frac{\partial^2 u}{\partial x \partial y}$ exists in \mathbb{R}^2 .