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Boundary and Eigenvalue Problems: 7th problem sheet

Exercise 1

Let $u(x) = 1 - |x|$ for $x \in I := (-1, 1)$.

a) Prove that $u \in W^{1,\infty}(I)$.

b) Show that $u \in W_0^{1,p}(I)$ for all $p \in [1, \infty)$ but $u \notin W_0^{1,\infty}(I)$.

c) Show that u is not a weak solution of the boundary value problem

$$u'' = 0 \quad \text{in } I\,, \qquad u(-1) = u(1) = 0$$

although it satisfies the boundary conditions and solves the differential equation almost everywhere.

Exercise 2

Let $\Omega \subset \mathbb{R}^n$ be a bounded domain and $c \in L^\infty(\Omega), c \geq 0, f \in L^2(\Omega)$.

a) Prove that the norm $|||u||| := (\int_\Omega |\nabla u|^2 + c(x)u^2 \, dx)^{\frac{1}{2}}$ is equivalent to the usual $W_0^{1,2}(\Omega)$-norm.

b) Show that the boundary value problem

$$\begin{aligned} -\Delta u + c(x)u &= f \quad \text{in } \Omega \\ u &= 0 \quad \text{on } \partial\Omega \end{aligned}$$

has a unique weak solution in $W_0^{1,2}(\Omega)$.