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Classical Methods for Partial Differential Equations

Exercise sheet 13

Exercise 45

Let $\Omega = \{(x,y) \in \mathbb{R}^2 \colon x > 0,\ y > 0\}$. Determine the type of the following differential equations and transform them into their normal form

1. $\frac{\partial^2 u}{\partial x^2} - 2\frac{\partial^2 u}{\partial x \partial y} + (1-x^2)\frac{\partial^2 u}{\partial y^2} = 0\,((x,y) \in \Omega)$,
2. $\frac{\partial^2 u}{\partial x^2} + y\frac{\partial^2 u}{\partial y^2} = 0\,((x,y) \in \Omega)$.

Exercise 46

Let $\Omega = \{(x,y) \in \mathbb{R}^2 \colon x > 0,\ y > 0\}$. Determine the general solution of the differential equation

$$x\frac{\partial^2}{\partial x^2} + (y-x)\frac{\partial^2 u}{\partial x \partial y} - y\frac{\partial^2 u}{\partial y^2} = 0 \quad ((x,y) \in \Omega)\,.$$

Exercise 47

Let $\Omega \subseteq \mathbb{R}^n$ be a domain and $\widetilde{a} \in \mathcal{C}^1(\Omega \times \Omega, \mathbb{R})$, $\gamma \in \mathcal{C}(\Omega \times \Omega \times \mathbb{R}, \mathbb{R})$ and $\sigma \in \{-1, 1\}$. Transform the differential equation

$$\frac{\partial^2 u}{\partial x^2} = \widetilde{a}(x,y)\frac{\partial u}{\partial x} + \sigma\frac{\partial u}{\partial y} + \gamma(x,y,u)\,,$$

into the normal form

$$\frac{\partial^2 v}{\partial x^2} = \sigma\frac{\partial v}{\partial y} + \widetilde{\gamma}(x,y,v)\,.$$

Hint: Use a transformation of the form $u = vw$, where w is a positive, smooth function.

Above exercises will be discussed on 03.02.2016.