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Classical Methods for Partial Differential Equations

Exercise sheet 14

Exercise 50

Let $\Omega = \big\{(x, y) \in \mathbb{R}^2 \colon x > 0,\ y > 0\big\}$. Determine the type of the following differential equations and transform them into their normal form

1. $\frac{\partial^2 u}{\partial x^2} - 2\frac{\partial^2 u}{\partial x \partial y} + \left(1 - x^2\right) \frac{\partial^2 u}{\partial y^2} = 0\,((x, y) \in \Omega)$,
2. $\frac{\partial^2 u}{\partial x^2} + y\frac{\partial^2 u}{\partial y^2} = 0\,((x, y) \in \Omega)$.

Exercise 51

Let $\Omega = \big\{(x, y) \in \mathbb{R}^2 \colon x > 0,\ y > 0\big\}$. Determine the general solution of the differential equation

$$x\frac{\partial^2 u}{\partial x^2} + (y - x)\frac{\partial^2 u}{\partial x \partial y} - y\frac{\partial^2 u}{\partial y^2} = 0 \quad ((x, y) \in \Omega)\,.$$

Exercise 52

Let $\Omega = (0, \infty)^2$.

1. Determine the type of the following partial differential equation, reduce it to its normal form and obtain its general solution

$$x^2\frac{\partial^2 u}{\partial x^2}(x, y) + 2xy\frac{\partial^2 u}{\partial x \partial y}(x, y) + y^2\frac{\partial u^2}{\partial y^2}(x, y) = 4x^2 \quad ((x, y) \in \Omega)\,.$$

2. Write the equation

$$x^2\frac{\partial^2 u}{\partial x^2}(x, y) + 2xy\frac{\partial^2 u}{\partial x \partial y}(x, y) + y^2\frac{\partial u^2}{\partial y^2}(x, y) = 4x^2 + x^4\frac{\partial u}{\partial x} \quad ((x, y) \in \Omega)\,.$$

in the form

$$\frac{\partial^2 u}{\partial \xi^2} = \frac{\partial u}{\partial \eta} + \gamma(\xi, \eta, u)\,.$$