Karlsruher Institut für Technologie (KIT)
Institut für Analysis
Prof. Dr. Michael Plum
M. Sc. Peter Rupp

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Nonlinear Boundary Value Problems

Exercise sheet 13

Exercise 39:

Check if the following mappings satisfy the Palais-Smale condition

(a) $f : \mathbb{R} \to \mathbb{R},\ x \mapsto x\ \sin(x)$

(b) $f : \mathbb{R}^2 \to \mathbb{R},\ (x, y) \mapsto \sin(x) + xy^2$

(c) $I : L^2((-1, 1)) \to \mathbb{R},\ u \mapsto \int_0^1 u(t)^2\, dt$

(d) $I : H_0^1((-1, 1)) \to \mathbb{R},\ u \mapsto \int_{-1}^1 u'(t)^2\, dt + \int_0^1 u(t)^2\, dt$

(e) $I : H_0^1((-1, 1)) \to \mathbb{R},\ u \mapsto \int_0^1 u'(t)^2\, dt + \int_0^1 u(t)^2\, dt$

(f) $I : H_0^1(\Omega) \to \mathbb{R},\ u \mapsto \int_\Omega \left[\frac{1}{2}|\nabla u|^2 - \frac{1}{4}|u|^4\right] dx$, where $\Omega \subseteq \mathbb{R}^2$ is a bounded domain.

Exercise 40:

(a) Let $f \in C^1(\mathbb{R}^n, \mathbb{R})$, Show that f satisfies the Palais-Smale condition provided that $|f| + |f'|$ is coercive.

(b) Let $f \in C^1(\mathbb{R}^n, \mathbb{R})$ be bounded from below and satisfy the Palais-Smale condition. Show that f is coercive and attains its minimum.

Exercise 41:

Let $(X, \|\cdot\|)$ be a normed space and let $L : X \to \mathbb{R}$ be a continously Fréchet-differentiable and convex (i.e. $L[(1-t)x + ty] \leq (1-t)L[x] + tL[y]$ $(x, y \in X,\ t \in [0, 1])$) mapping. Prove the following statements:

(a) $L[x] - L[y] \leq L[x](x - y)$ $(x, y \in X)$

(b) If L is strictly convex, then L has at most one critical value.

(c) Give an example of a strictly convex functional without critical points.