

Nonlinear Boundary Value Problems

Exercise sheet 14

Exercise 42:

Let $f : \mathbb{R}^2 \rightarrow \mathbb{R}$ defined by

$$f(x, y) = 9(x^2 + y^2) - (x^2 + y^2)^2 + y \sin(x).$$

Use the mountain pass theorem to show that f has a strictly positive critical value.

Exercise 43:

Let $L > 0$ and $\Omega = (-L, L) \subset \mathbb{R}$. Consider the following boundary value problem

$$\begin{cases} -u'' + (u + 1)^+ - u - 1 = 0 & \text{in } \Omega, \\ u(-L) = u(L) = 0, \end{cases}$$

where $y^+ := \max\{0, y\}$ for $y \in \mathbb{R}$. Define the corresponding functional and prove the existence of a non-trivial critical point.