

Aspects of nonlinear wave equations

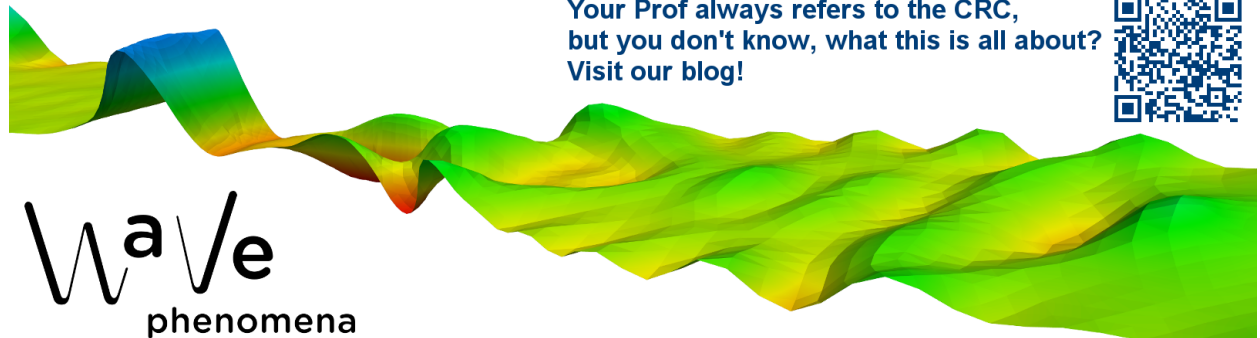
Problem 1 Sheet 11

Assume $m > 0$. In this problem we extend the results of Section 4.2 to standing waves of


$$\left\{ \begin{array}{ll} u_{tt} + u_{xxxx} + mu = \pm u^3 & ((x, t) \in (0, \pi) \times \mathbb{R}), \\ u(0, t) = u(\pi, t) = 0 & (t \in \mathbb{R}), \\ u_{xx}(0, t) = u_{xx}(\pi, t) = 0 & (t \in \mathbb{R}), \\ u(x, t + T) = u(x, t) & ((x, t) \in (0, \pi) \times \mathbb{R}), \\ u(x, t) = u(x, -t) & ((x, t) \in (0, \pi) \times \mathbb{R}). \end{array} \right.$$


Notice that now there are four boundary conditions – they are called Navier-conditions.

- (a) Find the free vibrations of the operator $L = \partial_t^2 + \partial_x^4 + m$ under the above Navier- and periodicity conditions. *Hint:* first solve the eigenvalue problem $\psi'''' + m\psi = \lambda\psi$ on $(0, \pi)$ with $\psi(0) = \psi''(0) = \psi(\pi) = \psi''(\pi) = 0$. The frequencies $(\omega_l)_{l \in \mathbb{N}}$ of the free vibrations with $\omega_1 < \omega_2 < \dots$ are related to the eigenvalues $(\lambda_l)_{l \in \mathbb{N}}$ with $\lambda_1 < \lambda_2 < \dots$
- (b) For $\omega > 0$ consider the operator $L_\omega = \omega^2 \partial_t^2 + \partial_x^4 + m$ with Navier-conditions at $x = 0, x = \pi$, 2π -periodicity and evenness in t . Fix a free-vibration frequency ω_{l_0} from (a) and assume $m \in \mathbb{R} \setminus \mathbb{Q}$. Show that $K_{l_0} := \text{Ker } L_{\omega_{l_0}} = \text{span}\{\psi_{l_0}(x) \cos t\}$.
- (c) Let $m \in \mathbb{R} \setminus \mathbb{Q}$ and ω be such that $|\omega - \omega_1| \leq \frac{\omega_1}{4}$ with ω_1 from (a). Find a γ -nonresonance condition for ω such that L_ω has a bounded inverse $L_\omega^{-1} : \mathcal{H} \cap K_1^\perp \rightarrow \mathcal{H} \cap K_1^\perp$ for a suitable Hilbert-space \mathcal{H} . Is this nonresonance condition covered by the Assumption of Section 4.2? *Hints:* \perp is with respect to $L^2(D)$ for $D = (0, \pi) \times (0, 2\pi)$. Define $\mathcal{H} \subset H^{4,2}(D)$ where $H^{4,2}(D)$ stands for functions v with $v_{xxxx}, v_{tt} \in L^2(D)$.



**Your Prof always refers to the CRC,
 but you don't know, what this is all about?
 Visit our blog!**





<https://blog.waves.kit.edu/wp/>

Recall:
 Oral exams dates: 08.09.2016 (Thursday), 07.10.2016 (Friday)