

## Aspects of nonlinear wave equations

### Sheet 12

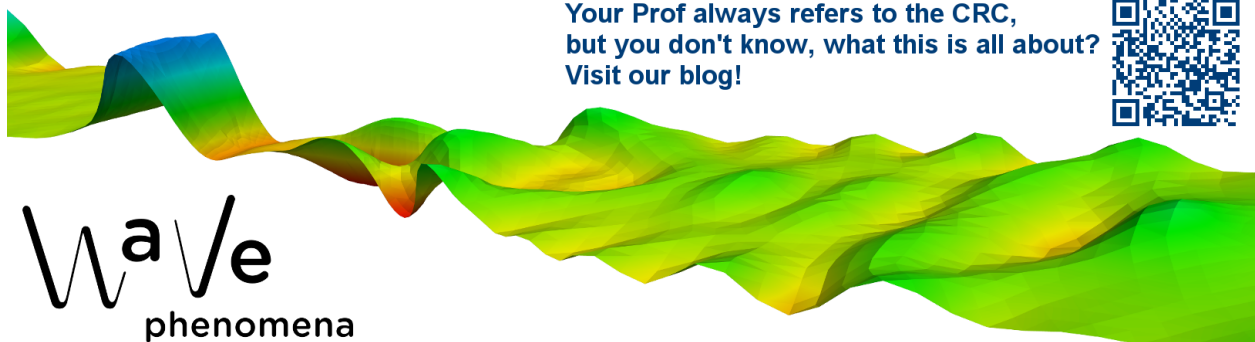
#### Problem 1

We use the notation from Sheet 9/Sheet 10. Assume  $r \in \mathbb{N}$ ,  $r \geq 2$ ,  $\alpha, \epsilon \in \mathbb{R}$  and  $h \in \mathcal{C}^{1,\alpha}(\bar{D})$  with  $D = (-\frac{\pi}{2}, \frac{\pi}{2}) \times (0, \pi)$ . In this problem the idea is to use the implicit function theorem once again to find solutions of

$$\begin{cases} u_{tt} - u_{xx} = \alpha u^r + \epsilon h(x, t) & ((x, t) \in (-\frac{\pi}{2}, \frac{\pi}{2}) \times \mathbb{R}), \\ u(-\frac{\pi}{2}, t) = u(\frac{\pi}{2}, t) = 0 & (t \in \mathbb{R}), \\ u(x, t + \pi) = u(x, t) & ((x, t) \in (-\frac{\pi}{2}, \frac{\pi}{2}) \times \mathbb{R}), \\ u(-x, t) = u(x, t) = u(x, -t) & ((x, t) \in (-\frac{\pi}{2}, \frac{\pi}{2}) \times \mathbb{R}). \end{cases} \quad (\star)$$

- With the help of the operator  $K$  from Sheet 9/Sheet 10 rewrite  $(\star)$  as  $u = K(\alpha u^r + \epsilon h)$  for  $u \in \mathcal{C}$ . Here  $\mathcal{C} = \{\psi \in C([-\frac{\pi}{2}, \frac{\pi}{2}] \times \mathbb{R}) : \psi \text{ is even in } x \text{ and } t, \pi\text{-periodic in } t\}$  is a Banach space with the norm  $\|\cdot\|_\infty$ .
- Show that  $F : \mathcal{C} \times \mathbb{R} \mapsto \mathcal{C}$  defined by  $u \mapsto u - K(\alpha u^r + \epsilon h)$  is a  $C^1$ -map and compute its derivatives  $\partial_u F$ ,  $\partial_\epsilon F$ . Start with the simplest case  $r = 2$ .
- Show that there exists  $\epsilon_0 > 0$  such that  $(\star)$  has a classical solution  $u = u_\epsilon$  for every  $\epsilon \in (-\epsilon_0, \epsilon_0)$ .
- Find conditions on  $f : \bar{D} \times \mathbb{R} \rightarrow \mathbb{R}$  so that the same existence statement holds for

$$\begin{cases} u_{tt} - u_{xx} = f(x, t, u) + \epsilon h(x, t) & ((x, t) \in (-\frac{\pi}{2}, \frac{\pi}{2}) \times \mathbb{R}), \\ u(-\frac{\pi}{2}, t) = u(\frac{\pi}{2}, t) = 0 & (t \in \mathbb{R}), \\ u(x, t + \pi) = u(x, t) & ((x, t) \in (-\frac{\pi}{2}, \frac{\pi}{2}) \times \mathbb{R}), \\ u(-x, t) = u(x, t) = u(x, -t) & ((x, t) \in (-\frac{\pi}{2}, \frac{\pi}{2}) \times \mathbb{R}). \end{cases} \quad (\star\star)$$



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Recall:  
 Oral exams dates: 08.09.2016 (Thursday), 07.10.2016 (Friday)