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Aspects of nonlinear wave equations

Sheet 2

Problem 1 Consider the equation

$$u_{tt} - u_{xx} = |1 - u| - 1.$$

For every $\omega \in (-1, 1) \setminus \{0\}$ compute explicitly a traveling wave $u(x,t) = v(x - \omega t)$ such that

(a) $v(-s) = v(s)$ for all $s \in \mathbb{R}$,

(b) $v > 0$,

(c) $v(s) \xrightarrow[s \to \pm\infty]{} 0$.

Hint: Solve the equation for v with $v'(0) = 0$, $v(s) > 1$ on $[0, a)$, $0 < v(s) < 1$ on (a, ∞).

Problem 2 Consider the equation

$$u_{tt} - u_{xx} = (1 - u)^+ - 1,$$

where $\alpha^+ = \max\{0, \alpha\}$, $\alpha \in \mathbb{R}$. Show that it does not have any traveling wave $u(x,t) = v(x - \omega t)$ such that $v(s) \xrightarrow[s \to \pm\infty]{} 0$.

Does it have any periodic traveling wave?