

Aspects of nonlinear wave equations

Sheet 4

Problem 1 Consider the wave equation

$$u_{tt} - u_{xx} = f(u) \text{ in } \mathbb{R} \times \mathbb{R}.$$

Give explicit examples for the nonlinearity $f: \mathbb{R} \rightarrow \mathbb{R}$ such that one of the following holds:

- (a) for every wavespeed $\omega \in (-1, 1) \setminus \{0\}$ there are exactly two homoclinic profiles for traveling waves and all other traveling waves are periodic.
- (b) for every wavespeed $\omega \in (-1, 1) \setminus \{0\}$ there is exactly one homoclinic profile, exactly two heteroclinic profiles and infinitely many periodic traveling waves.

Draw a sketch of the phase plane for the profiles.

Problem 2 Compute in closed form all traveling waves for Burger's equation

$$u_t + uu_x - au_{xx} = 0 \text{ in } \mathbb{R} \times \mathbb{R},$$

where $a \in \mathbb{R}$ is a given constant. How does the phase plane for the profiles look like?