P 10.1 $\mathcal{C}^1 = \mathcal{C}^{1,0}$ 05.07.2016 (d)

(a) $$C(t) = \frac{1}{4}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\int_{t-\frac{\pi}{2}-y}^{t+\frac{\pi}{2}+y} f(y,s)\,ds\,dy \qquad C'(t) = \frac{1}{4}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} f(y,t+\tfrac{\pi}{2}+y) - f(y,t-\tfrac{\pi}{2}-y)\,dy$$

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} f(y,t-\tfrac{\pi}{2}-y)\,dy \overset{f(a,b)=f(-a,b)}{=} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} f(-y,t-\tfrac{\pi}{2}-y)\,dy \underset{ds=-dy}{\overset{s=-y}{=}} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} f(s,t-\tfrac{\pi}{2}+s)\,ds$$

$$\overset{f(a,b)=f(a,b+\pi)}{=} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} f(y,t+\tfrac{\pi}{2}+y)\,dy \Rightarrow C'(t)=0 \Rightarrow C \text{ is constant.}$$

$$u(-\tfrac{\pi}{2},t) = \frac{1}{4}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\int_{t-\frac{\pi}{2}-y}^{t+\frac{\pi}{2}+y} f(y,s)\,ds\,dy - \frac{1}{2}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\int_{t-\frac{\pi}{2}-y}^{t+\frac{\pi}{2}+y} f(y,s)\,ds\,dy + c = \frac{1}{4} - \frac{1}{2} + \frac{1}{4} = 0$$

$$u(\tfrac{\pi}{2},t) \underset{\uparrow}{=} u(-\tfrac{\pi}{2},t) = 0$$

u IS EVEN IN SPACE

$$f \text{ even in } t \Rightarrow \int_{-t+(x-y)}^{-t-(x-y)} f(y,s)\,ds = \int_{t+(x-y)}^{t-(x-y)} f(y,s)\,ds \Rightarrow u \text{ IS EVEN IN TIME}$$

$$f \text{ even in } x \Rightarrow \int_a^b \int_{t-x-y}^{t+x+y} f(y,s)\,ds = \int_a^b F_2(y,t+x+y) - F_2(y,t-x-y)\,dy \overset{\xi=-y}{=} \int_{-b}^{-a} \underbrace{F_2(-\xi,t+x-\xi)}_{=F_2(\xi,\dots)} - \underbrace{F_2(-\xi,t-x+\xi)}_{=F_2(\xi,\dots)}\,d\xi$$

$$= \int_{-b}^{-a} F_2(y,t+x-y) - F_2(y,t-x+y)\,dy = \int_{-b}^{-a}\int_{t-x+y}^{t+x-y} f(y,s)\,ds\,dy \qquad (*)$$

$$u(-x,t) = \frac{1}{4}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\int_{t-x-y}^{t+x+y} f(y,s)\,dy\,ds - \frac{1}{2}\int_{-x}^{\frac{\pi}{2}}\int_{t-x-y}^{t+x+y} f(y,s)\,dy\,ds + c \overset{(*)}{=} \frac{1}{4}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\int_{t-x+y}^{t+x-y} f(y,s)\,ds + \frac{1}{2}\int_{X}^{-\frac{\pi}{2}}\int_{t-x+y}^{t+x-y} f(y,s)\,ds$$

$$= u(x,t) + \frac{1}{2}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\int_{t-x+y}^{t+x-y} f(y,s)\,dy\,ds + \underbrace{\frac{1}{2}\int_{X}^{\frac{\pi}{2}}\int_{t+x-y}^{t-x+y} f(y,s)\,dy\,ds - \frac{1}{2}\int_{X}^{-\frac{\pi}{2}}\int_{t+x-y}^{t-x+y} f(y,s)\,dy\,ds}_{=\frac{1}{2}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\int_{t+x-y}^{t-x+y} f(y,s)\,ds\,dy} \overset{(*)}{=} u(x,t)$$

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(b)

$$\varphi(x,t)=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\int_{t+x-y}^{t-x+y} f(y,s)\,ds\,dy \qquad \psi(x,t)=\int_{x}^{\frac{\pi}{2}}\int_{t+x-y}^{t-x+y} f(y,s)\,ds\,dy$$

$$v(x,t)=\frac{1}{4}\varphi(x,t)-\frac{1}{2}\psi(x,t)+C \qquad C=\frac{1}{4}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\int_{t-\frac{\pi}{2}-y}^{t+\frac{\pi}{2}+y} f(y,s)\,ds\,dy=\frac{1}{4}\psi\left(-\frac{\pi}{2},t\right)$$

$$=\frac{1}{4}\varphi\left(-\frac{\pi}{2},t\right)$$

$$\frac{d}{dz}\int_{a(z)}^{b(z)} h(\xi,z)\,d\xi=\int_{a(z)}^{b(z)} h'_z(\xi,z)\,d\xi+h(b(z),z)\,b'(z)-h(a(z),z)\cdot a'(z)$$

$$\frac{\partial\varphi}{\partial x}=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} -f(y,t-x+y)-f(y,t+x-y)\,dy, \quad \frac{\partial\varphi}{\partial t}=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} f(y,t-x+y)-f(y,t+x-y)\,dy,$$

$$\frac{\partial^2\varphi}{\partial x^2}=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} f'_2(y,t-x+y)-f'_2(y,t+x-y), \quad \frac{\partial^2\varphi}{\partial t^2}=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} f'_2(y,t-x+y)-f'_2(y,t+x-y)\,dy,$$

$$\frac{\partial\psi}{\partial x}=\int_{x}^{\frac{\pi}{2}} -f(y,t-x+y)-f(y,t+x-y)\overbrace{-F_2(x,t-x+x)+F_2(x,t+x-x)}^{=0}$$

$$\frac{\partial^2\psi}{\partial x^2}=\int_{x}^{\frac{\pi}{2}} f'_2(y,t-x+y)-f'_2(y,t+x-y)+\overbrace{f(x,t-x+x)+f(x,t+x-x)}^{=2f(x,t)}$$

$$\frac{\partial\psi}{\partial t}=\int_{x}^{\frac{\pi}{2}} f(y,t-x+y)-f(y,t+x-y)\,dy \qquad \frac{\partial^2\psi}{\partial t^2}=\int_{x}^{\frac{\pi}{2}} f'_2(y,t-x+y)-f'_2(y,t+x-s)\,dy$$

$$Lv=v_{tt}-v_{xx}=\frac{1}{4}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\sqcup-\frac{1}{2}\int_{x}^{\frac{\pi}{2}}\sqcup-\frac{1}{4}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\sqcup+\frac{1}{2}\int_{x}^{\frac{\pi}{2}}\sqcup+f(x,t)=f(x,t)$$

ASSUMING THAT
$C =$ CONSTANT

$$\frac{\partial^2\varphi}{\partial t\partial x}=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} -f'_2(y,t-x+y)-f'_2(y,t+x-y)\,dy$$

$$\frac{\partial^2\psi}{\partial t\partial x}=\int_{x}^{\frac{\pi}{2}} -f'_2(y,t-x+y)-f'_2(y,t+x-y)\,dy$$

P.10.1 (b) $\int_a^b h\,ds - \int_c^d h\,ds = \int_a^c + \int_d^b h\,ds = \int_a^c h\,ds + \int_d^b h\,ds$

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$$\bullet \ |\psi(x,t_1) - \psi(x,t_2)| = \left| \int_x^{\frac{\pi}{2}} \int_{t_1+x-y}^{t_1-x+y} f(y,s)\,ds - \int_{t_2+x-y}^{t_2-x+y} f(y,s)\,ds\,dy \right|$$

$$\leq \int_0^{\frac{\pi}{2}} \left| \int_{t_1+x-y}^{t_2+x-y} f(y,s)\,ds \right| + \left| \int_{t_2-x+y}^{t_1-x+y} f(y,s)\,ds \right| dy \leq \int_0^{\frac{\pi}{2}} |t_1-t_2|^{1-\frac{1}{q}} \left( \int_{t_1+x-y}^{t_2+x-y} |f(y,s)|^q\,ds \right)^{\frac{1}{q}} dy$$

$$+ \int_0^{\frac{\pi}{2}} |t_1-t_2|^{1-\frac{1}{q}} \left( \int_{t_2-x+y}^{t_1-x+y} |f(y,s)|^q\,ds \right)^{\frac{1}{q}} dy$$

$$\leq \pi\, |t_1-t_2|^{1-\frac{1}{q}} \, \|f\|_{L^q(D)}$$

$$\bullet \ |\psi(x_1,t) - \psi(x_2,t)| = \left| \int_{x_1}^{\frac{\pi}{2}} \int_{t+x_1-y}^{t-x_1+y} f(y,s)\,ds\,dy - \int_{x_2}^{\frac{\pi}{2}} \int_{t+x_2-y}^{t-x_2+y} f(y,s)\,ds\,dy \right| \leq$$

$$\leq \int_{x_1}^{x_2} \int_{t+x_1-y}^{t-x_1+y} |f(y,s)|\,ds\,dy + \int_{x_2}^{\frac{\pi}{2}} \left| \int_{t+x_1-y} f(y,s)\,ds - \int_{t+x_2-y} f(y,s)\,ds \right| dy$$

$$= \int_{x_1}^{x_2} \int_{t+x_1-y}^{t-x_1+y} |f(y,s)|\,ds\,dy + \int_{x_2}^{\frac{\pi}{2}} \left| \int_{t+x_1-y}^{t+x_2-y} f(y,s)\,ds + \int_{t-x_2+y}^{t-x_1+y} f(y,s)\,ds \right| dy$$

$$\leq |x_1-x_2|^{1-\frac{1}{q}} \|f\|_{L^q} \pi^{1-\frac{1}{q}} + |x_1-x_2|^{1-q} \|f\|_{L^q} \pi^{1-\frac{1}{q}}$$

$$\bullet \ |\varphi(x,t_1) - \varphi(x,t_2)| = \left| \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_{t_1+x-y}^{t_1-x+y} f(y,s)\,ds - \int_{t_2+x-y}^{t_2-x+y} f(y,s)\,ds \right| \leq \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left| \int_{t_1+x-y}^{t_2+x-y} f(y,s)\,ds \right| + \left| \int_{t_2-x+y}^{t_1-x+y} f(y,s)\,ds \right| dy$$

$$\leq \pi \cdot |t_1-t_2|^{1-\frac{1}{q}} \|f\|_{L^q}$$

$$\bullet \ |\varphi(x_1,t) - \varphi(x_2,t)| \leq \ldots \leq C\, |\cancel{x}_1 - x_2|^{1-\frac{1}{q}} \|f\|_{L^q}$$

$$\bullet \ |u(x_1,t_1) - u(x_2,t_2)| \leq |u(x_1,t_1) - u(x_2,t_1)| + |u(x_2,t_1) - u(x_2,t_2)|$$

$$\leq C_1 |x_1-x_2|^{\alpha} + C_2 |t_1-t_2|^{\alpha} \leq C \left\| \begin{pmatrix} x_1 \\ t_1 \end{pmatrix} - \begin{pmatrix} x_2 \\ t_2 \end{pmatrix} \right\|^{\alpha}$$

$\Rightarrow K : \mathcal{L}^q \to \mathcal{C}^{0,\alpha}$ $(q \in (1,\infty))$

$K \in \mathcal{L}(\mathcal{L}^q, \mathcal{C}^{0,\alpha})$

IF $f \in \mathcal{C}^{0,\alpha} \Rightarrow \frac{\partial \varphi}{\partial x_1}, \frac{\partial \varphi}{\partial t}, \frac{\partial \psi}{\partial x_1}, \frac{\partial \psi}{\partial t} \in \mathcal{C}^{0,\alpha} \Rightarrow v \in \mathcal{C}^{1,\alpha}$

IF $f \in \mathcal{C}^{1,\alpha} \Rightarrow \frac{\partial^2 \varphi}{\partial x^2}, \ldots, \frac{\partial^2 \psi}{\partial t^2} \in \mathcal{C}^{0,\alpha} \Rightarrow v \in \mathcal{C}^{2,\alpha}$