P12.1 18 07 2016 ⓐ

a) $u = K(\alpha u^\nu + \varepsilon h) \quad u \in \mathcal{C} \subseteq \mathcal{L}_q \quad K : \mathcal{L}_q \to \mathcal{C}^{0,\alpha}$

b) $F : \mathcal{C} \times \mathbb{R} \to \mathcal{C} \quad F(u,\varepsilon) = u - K(\alpha u^\nu + \varepsilon h).$

- $u \in \mathcal{C} \to u^\nu \in \mathcal{C}$ (CONTINUITY, π-PERIODIC in t, even in $x_1(t)$).

- $F(0,0) = 0$

- $\frac{\partial F}{\partial u}(u,\varepsilon)(\varphi) = \varphi - K(\alpha \nu u^{\nu-1}\varphi)$

$$(u+\varphi)^\nu - u^\nu - \nu u^{\nu-1}\varphi = \int_0^1 \frac{d}{dt}(u+t\varphi)^\nu dt - \nu u^{\nu-1}\varphi$$

$$= \int_0^1 \nu\left((u+t\varphi)^{\nu-1} - u^{\nu-1}\right)\varphi\, dt$$

$$= \int_0^1\int_0^1 \nu \frac{d}{ds}(u+ts\varphi)^{\nu-1}\varphi\, ds\, dt = \nu(\nu-1)\int_0^1\int_0^1 (u+ts\varphi)^{\nu-2} s\varphi^2\, ds\, dt$$

$\xi \mapsto \xi^{\nu-2}$ is CONVEX

$$|(u+\varphi)^\nu - u^\nu - \nu u^{\nu-1}\varphi| \leq C\|\varphi\|^2 \|(|u|+|\varphi|)^{\nu-2}\|_\infty$$
$$\leq C 2^{\nu-2}\|\varphi\|_\infty^2 (\|u\|_\infty^{\nu-2} + \|\varphi\|_\infty^{\nu-2})$$

Hence $\lim_{\varphi \xrightarrow{\mathcal{C}} 0} \frac{\|(u+\varphi)^\nu - u^\nu - \nu u^{\nu-1}\varphi\|_\infty}{\|\varphi\|_\infty} = 0$

- $u \mapsto \frac{\partial F}{\partial u}(u,\varepsilon)$ is CONTINOUS (cf. later))

- $\frac{\partial F}{\partial \varepsilon}(u,\varepsilon)\eta = -(Kh)\eta \quad \left\{\frac{\partial F}{\partial \varepsilon}(u,\varepsilon) : \mathbb{R} \to \mathcal{C}\right\}$

$\cong -Kh$

(c) $\frac{\partial F}{\partial u}(u,\varepsilon) = Id - K(\alpha \nu u^{\nu-1}\,\cdot\,)$

$\frac{\partial F}{\partial u}(0,0) = Id \Rightarrow \frac{\partial F}{\partial u}(0,0)$ IS INVERTIBLE.

BY IFT $\exists_{\varepsilon_0 > 0}\ u : (-\varepsilon_0, \varepsilon_0) \to \mathcal{C} : F(u(\varepsilon),\varepsilon) = 0 \quad u(0) = 0$

$$\left.\begin{aligned} &u = K(\alpha u^\nu + \varepsilon h) \hookrightarrow u \in \mathcal{C}^{0,\alpha}(\bar{D}) \\ &u^\nu \in \mathcal{C}^{0,\alpha}(\bar{D}) \hookrightarrow u \in \mathcal{C}^{1,\alpha}(\bar{D}) \hookrightarrow u^\nu \in \mathcal{C}^{1,\alpha}(\bar{D})\end{aligned}\right\} \text{cf. 05.07.2016 (f)}$$

$u \in \mathcal{C}^{2,\alpha}(\bar{D})$ is a CLASSICAL SOLUTION OF $(*)$

P.12.1 (d) 18.07.2016 (6)

ASSUMPTIONS ON f: $f(x,t,s)$

(f_1) $f(x,t,0)=0$, $f\in C^1([-\frac{\pi}{2},\frac{\pi}{2}]\times\mathbb{R}\times\mathbb{R})$

(f_2) $\frac{\partial f}{\partial s}(x,t,0)=0$ $f(x,t+\pi,s)=f(x,t,s)$

$f(-x,t,s)=f(x,t,s)=f(x,-t,s)$

(f_3) $\frac{\partial f}{\partial x},\frac{\partial f}{\partial t},\frac{\partial f}{\partial s}\in C^{0,\alpha}([-\frac{\pi}{2},\frac{\pi}{2}]\times[0,\pi]\times[-M,M])$ $(M>0)$

NOTE: $(f_3)\Rightarrow f\in C^{0,\alpha}([-\frac{\pi}{2},\frac{\pi}{2}]\times[0,\pi]\times[-M,M])$. $(M>0)$

1) $u\in C\Rightarrow f(\cdot,\cdot,u(\cdot,\cdot))\in C$

2) $u\in C^{k,\alpha}\Rightarrow f(\cdot,\cdot,u(\cdot,\cdot))\in C^{k,\alpha}$ $k=0,1$

3) $\frac{\partial F}{\partial u}(u,\varepsilon)$ exists & is contins in (u,ε)

ad 3) Let $\|\varphi\|_\infty\le 1$, $M=\|u\|_\infty+1$

$\frac{\partial f}{\partial s}$ is uniformly cont on $[-\frac{\pi}{2},\frac{\pi}{2}]\times\mathbb{R}\times[-M,M]$ $\Rightarrow u\mapsto\frac{\partial F}{\partial u}$ is of cont

$f(x,t,u(x,t)+\varphi(x,t))-f(x,t,u(x,t))-f_s(x,t,u(x,t))\varphi(x,t)$

$$=\int_0^1 \underbrace{f_s(x,t,u(x,t)+\tau\varphi(x,t))-f_s(x,t,u(x,t))}_{\le\varepsilon \text{ if } \|\varphi\|_\infty\le\delta}\varphi(x,t)\,d\tau$$

Hence $\frac{\|f(\cdot,\cdot,u+\varphi)-f(\cdot,\cdot,u)-f_s(\cdot,\cdot,u)\varphi\|_\infty}{\|\varphi\|_\infty}\le\varepsilon$ IF $\|\varphi\|_\infty\le\delta$

ad (2) $k=0$, $M:=\|u\|_\infty$; $k=1$ Analog

$$|f(x_1,t_1,u(x_1,t_1))-f(x_2,t_2,u(x_2,t_2))|\le|f(x_1,t_1,u(x_1,t_1))-f(x_2,t_1,u(x_1,t_1))|$$

$$+|f(x_2,t_1,u(x_1,t_1))-f(x_2,t_2,u(x_1,t_1))|+|f(x_2,t_2,u(x_1,t_1))-f(x_2,t_2,u(x_2,t_2))|$$

$$\le C_1|x_1-x_2|^\alpha+C_2|t_1-t_2|^\alpha+C_3|u(x_1,t_1)-u(x_2,t_2)|^\alpha$$

$$\le C_1|x_1-x_2|^\alpha+C_2|t_1-t_2|^\alpha+C_3\underbrace{|(x_1,t_1)-(x_2,t_2)|^\alpha}_{\le C_4|(x_1,t_1)-(x_2,t_2)|^\alpha}$$