P.2.1 28.04.2016 (a)

$$u_{tt} - u_{xx} = |1-u| - 1 \qquad (\bullet)$$

INSERTING $u(x,t) = v(x-wt)$ INTO $(\bullet)$ GIVES:

$$w^2 v''(x-wt) - v''(x-wt) = |1-v| - 1$$

$$(w^2-1)\, v'' = |1-v| - 1 \qquad (\bullet\bullet)$$

IF p SOLVES $(*)$, THEN

$$v(s) = p\left(\frac{s}{\sqrt{1-w^2}}\right)$$

$$v''(s) = \frac{1}{w^2-1}\, p''\left(\frac{s}{\sqrt{1-w^2}}\right)$$

SOLVES $(\bullet\bullet)$

CONSIDER $w \in (-1,1)\setminus\{0\}$ and

$$-p'' = |1-p| - 1 = \begin{cases} p-2, & p>1 \\ -p & p \le 1 \end{cases} \qquad (*)$$

SOLVE $(*)$ WITH $p'(0)=0$, $p>1$ on $[0,a)$ FOR SOME $a>0$ and $0<p<1$ on (a,∞)

JUST ON $[0,\infty)$! WE WANT EVEN SOLUTIONS

- IF $p>1$: $(*)$ BECOMES

$$-p'' = p-2$$

SOLUTION $p(s) = 2 + \alpha\cos(s) \quad (\alpha\in\mathbb{R})$

$p'(s) = -\alpha \sin s$

- IF $0<p<1$ $(*)$ BECOMES

$$-p'' = -p$$

$$p(s) = \beta e^{-s} \qquad (\beta\in\mathbb{R}) \qquad p'(s) = -\beta e^{-s}$$

FOR $a\in\mathbb{R}$ FIND α,β IN SUCH A WAY THAT $p(a)=1$ & p IS $\mathcal{C}^1$.

$$\begin{cases} 2+\alpha\cos a = 1, & \beta e^{-a} = 1 \\ -\alpha\sin a = -\beta e^{-a} \end{cases}$$

$$\begin{cases} \alpha\cos a = -1 \\ \alpha\sin a = 1 \\ \beta = e^{a} \end{cases} \Rightarrow \quad tg\, a = -1 \quad \Rightarrow \quad a = \begin{cases} \frac{3}{4}\pi + k\pi & 1^\circ \\ \frac{7}{4}\pi + k\pi & 2^\circ \end{cases}$$

~~IN 1° $\cos a = \frac{\sqrt{2}}{2}$ $\sin a = -\frac{\sqrt{2}}{2}$ $\alpha = -\sqrt{2}$ $\Rightarrow p(s) = 2-\sqrt{2}\cos s$ $p(0) = 2-\sqrt{2} < 1$~~ ↯

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(b)

$\cancel{\sqrt{2}^0 \quad \cos a = -\frac{1}{2}\sqrt{2} \quad \sin a = \frac{1}{2}\sqrt{2} \quad \alpha = \sqrt{2}}$

Let $a = \frac{3}{4}\pi$

$p(s) = 2 + \sqrt{2}\cos s \qquad (s \in [0, a])$

NOTE THAT $\quad p(s) > 1 \quad (s \in [0, a))$ and $p(a) = 1 \quad a = \frac{3}{4}\pi$

HENCE:

$$P(x) = \begin{cases} 2 + \sqrt{2}\cos x & 0 \le x \le \frac{3}{4}\pi \\ e^{\frac{3}{4}\pi - x} & , \; x \geq \frac{3\pi}{4} \end{cases} \qquad (x \geq 0)$$

and $p(x) = p(-x) \quad (x < 0)$

$V(s) = p\left(\frac{s}{\sqrt{1-\omega^2}}\right)$

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2° $|\omega| > 1$

$$p'' = (1-p)^+ - 1 = \begin{cases} -p & , p < 1 \\ -1 & p > 1 \end{cases} \quad (\because)$$

IF p SOLVES $(\because) \Rightarrow$
$v(s) = p\left(\frac{s}{\sqrt{\omega^2-1}}\right), v''(s) = \frac{1}{\omega^2-1} p''\left(\frac{s}{\sqrt{\omega^2-1}}\right)$
SOLVES $(\cdot)$

$v(s) \xrightarrow[\pm\infty]{} 0 \iff p(s) \xrightarrow[s\to\pm\infty]{} 0 \quad :\Rightarrow$

$p(s) < 1 \quad (s \in (-\infty, a) \cup (b, \infty))$,

$p(s) = A\sin s + B\cos s \quad \lightning$

3° $|\omega| = 1$

$(\cdot)$ BECOMES $\quad (1-v)^+ - 1 = 0$

$$0 = \begin{cases} -v & v < 1 \\ -1 & v > 1 \end{cases}$$

$\Rightarrow \quad v < 1 \quad \Rightarrow \quad v = 0 \quad \lightning$

ARE THERE PERIODIC TRAVELING WAVES? YES.

Let $|\omega| > 1$ & let $v(s) = A\cos\left(\frac{s}{\sqrt{\omega^2-1}}\right) + B\sin\left(\frac{s}{\sqrt{\omega^2-1}}\right)$

$(A, B : v < 1)$

THEN v IS PERIODIC

v SOLVES $(\cdot)$

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$$(\omega^2-1)v'' = (1-v)^+ - 1 = \begin{cases} -v & v\le 1 \\ -1 & v>1 \end{cases} \quad (\circ)$$

1° $0<|\omega|<1$

$$-p'' = (1-p)^+ - 1 = \begin{cases} -p & ,p<1 \\ -1 & ,p>1 \end{cases} \quad (\circ\circ)$$

{ IF p SOLVES $(\circ\circ) \Rightarrow$ $v(s) = p\left(\frac{s}{\sqrt{1-\omega^2}}\right)$ $v''(s) = \frac{1}{\omega^2-1} p''\left(\frac{s}{\sqrt{1-\omega^2}}\right)$ SOLVES $(\circ)$ }

$v(s) \xrightarrow[s\to\pm\infty]{} 0 \iff p(s) \xrightarrow[s\to\pm\infty]{} 0$, SO.

$$p(s) < 1 \quad (s \in (-\infty, a) \cup (b,\infty))$$

$$p(s) = \beta_+ e^{-s} \quad (s\in(b,\infty))$$

$$p(s) = \beta_- e^{s} \quad (s \in (-\infty, a))$$

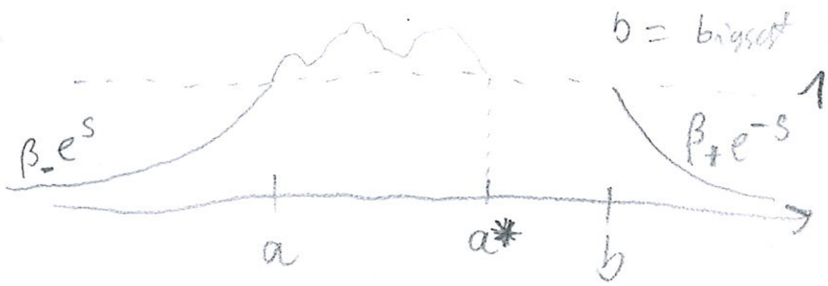
IF $\beta_+ \le 0 \Rightarrow p(s) \le 0 < 1 \; (s\in\mathbb{R}) \Rightarrow p(s) = \beta_+ e^{-s}$ ↯ $\Rightarrow \beta_+ > 0$

IF $\beta_- \le 0 \Rightarrow p(s) \le 0 < 1 \; (s\in\mathbb{R}) \Rightarrow p(s) = \beta_- e^{s}$ ↯ $\Rightarrow \beta_- > 0$

TAKE $\beta_+, \beta_- > 0$

a = SMALLEST VALUE $p(s)=1$

b = biggest VALUE $p(s)=1$



Let a^* = NEXT VALUE $p(s)=1$ $>a$ { MIGHT BE $a^*=b$ }

BETWEEN a, a^*: $-p''=-1$

$p''=1 \Rightarrow p$ IS CONVEX ↯

{ OR $p(s) = x^2 + \cancel{\gamma}x + \delta$ ↯ }