Travelling Waves, SS 2014
Exercise sheet 1

Exercise 1

(6 Points)

In the following, \( u : (x, t) \mapsto u(x, t) \) is a sufficiently smooth function.

a) **Klein-Gordon.** Let \( c, m \in \mathbb{R} \setminus \{0\} \). Show that the equation
\[
u_{tt} = c^2 \nu_{xx} - m^2 \nu \]
can be transformed into
\[
u_{tt} = \nu_{xx} - \nu.\]

b) **Sine-Gordon.** Show that the general form
\[
Au_{tt} - ku_{xx} = -T \sin(u)
\]
for \( A, T, k \in \mathbb{R}, T \neq 0 \) can be cast into
\[
u_{tt} - \nu_{xx} = -\sin(u).\]

Exercise 2

(6 Points)

Let \(|c| < 1\) and \( w(\xi) = v'(\xi) \) for \( \xi \in \mathbb{R} \). Consider the ordinary differential equation
\[
\begin{pmatrix} v' \\ w' \end{pmatrix}(\xi) = \begin{pmatrix} 0 & 1 \\ \frac{1}{1-c^2} & 0 \end{pmatrix} \begin{pmatrix} v' \\ w \end{pmatrix}(\xi), \quad \xi \in \mathbb{R},
\]
which is \( c^2 v'' = v'' - v \) for \( \xi \in \mathbb{R} \), written as a first order system.

Give a proof that the function \( v \) is bounded if and only if \( v(\xi) = 0 \) for all \( \xi \in \mathbb{R} \).

Exercise 3

(6 Points)

Have a look at the Korteweg-de Vries equation \( u_t = -uu_x - u_{xxx} \).

a) Check that
\[
v(\xi) = 3c \text{sech}^2 \left( \frac{\sqrt{c}}{2} \xi \right)
\]
for \( \xi \in \mathbb{R} \) is the profile of a travelling wave of the KdV equation with velocity \( c > 0 \).

b) Visualize the corresponding travelling wave solution in a suitable way. Use a programming language of your choice.

Deadline: Tuesday, April 29, at the beginning of the lecture.