Travelling Waves, SS 2014
Exercise sheet 4

Exercise 1 (6 Points)

Assume there exists $\delta > 0$, $k > 0$, such that for all $w_0 \in H^1$ with $\|w_0\|_{H^1} \leq \delta$ the solution $(w, \mu)$ of

\[
\begin{align*}
    w_t &= Pw + \mu w_x + q(w) + \mu w_x, \\
    0 &= \Psi(w + u) = (u', w), \\
    w(x, 0) &= u_0(x) - u(x) =: w_0(x),
\end{align*}
\]

exists for all $t \geq 0$ and there is $\eta > 0$, such that

\[
\begin{align*}
    \|w(\cdot, t)\|_{H^1} &\leq k \|w_0\|_{H^1} \quad \forall t \geq 0, \\
    \|w(\cdot, t)\|_{H^1} &\leq ke^{-\eta t} \quad \forall t \geq 0, \\
    |\mu(t)| &\leq ke^{-\eta t} \quad \forall t \geq 0, \\
    \int_0^\infty |\mu(\tau)| d\tau &\leq k \|w_0\|_{H^1}.
\end{align*}
\]

Show the travelling wave $(u, c)$ of $u_t = Au_{xx} + Bu_x + f(u)$ is asymptotically stable with asymptotic phase with respect to $H^1$-norm.

Deadline: Tuesday, July 1, at the beginning of the lecture.