Exercise 1 (6 Points)

Let \( f \in L^1_{\text{loc}}(\mathbb{R}^+, X) \), then define \( F(t) := \int_0^t f(s)ds \) and \( F_\infty := \lim_{t \to \infty} F(t) \) if the limit exists and \( F_\infty := 0 \) otherwise. Show

\[
\text{abs}(f) = \omega(F - F_\infty).
\]

You may use the following steps:

a) Suppose \( \text{abs}(f) < \infty \). Show \( \text{abs}(f) \geq \omega(F - F_\infty) \) for each of the cases

(i) \( \text{abs}(f) > 0 \),
(ii) \( \text{abs}(f) = 0 \),
(iii) \( \text{abs}(f) < 0 \).

b) Suppose \( \omega(F - F_\infty) < \infty \) and show \( \text{abs}(f) \leq \omega(F - F_\infty) \).

Exercise 2 (6 Points)

For \( t \geq 0 \) define \( f(t) = e^t e^t \cos(e^t) \). Then \( \omega(f) = \infty \) and show that \( \text{abs}(f) = 0 \).

Exercise 3 (6 Points)

Define \( f(t) = e^t \sin e^t \) for \( t \geq 0 \). Show that

\[
\text{hol}(\hat{f}) = -\infty < 0 = \text{abs}(f) < 1 = \omega(f).
\]

Discussion: Tuesday, November 24.