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1st Problem Sheet

Variational Methods and Applications to PDEs

Problem 1

Let Ω be a bounded, open subset of $\mathbb{R}^n$ and let $f : \Omega \times \mathbb{R} \longrightarrow \mathbb{R}$ be a continuous function satisfying

$$|f(x,s)| \leq C(1+|s|^{p-1}) \qquad \text{for all } (x,s) \in \Omega \times \mathbb{R}$$

for constants $C > 0$ and $p > 1$. Furthermore let $F : \Omega \times \mathbb{R} \longrightarrow \mathbb{R}$ be defined by

$$F(x,t) = \int_0^t f(x,s)ds \qquad \text{for all } (x,s) \in \Omega \times \mathbb{R}.$$

Prove that the functional $L : L^p(\Omega) \longrightarrow \mathbb{R}$ given by

$$L[u] = \int_\Omega F(x,u(x))dx, \qquad u \in L^p(\Omega)$$

is continuous.

Problem 2

Consider the functional

$$L[u] = \int_{-a}^{a} u(x)\sqrt{1+u'(x)^2}dx$$

for $u \in M := \{v \in C^1[-a,a] : v(a) = v(-a) = b\}$. Determine the Euler-Lagrange equation of L and solve it with boundary conditions $u(a) = u(-a) = b$.

To be discussed in the problem session on Tuesday, November 10, 2009.