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WS 2009/10

2^{nd} Problem Sheet

Variational Methods and Applications to PDEs

Problem 3

a) Let X be a normed space and let $f : \mathbb{R} \longrightarrow \mathbb{R}$ be a convex and monotonously increasing function. Prove that the functional $L : X \longrightarrow \mathbb{R}$ given by

$$L[u] = f(\|u\|) \qquad \text{for all } u \in X$$

is convex.

b) Show that there exist convex functionals which do not have minimizers.

Problem 4

Let X be a reflexive Banach space and $M \neq \emptyset$ a closed, convex subset of X.

a) Let $p > 1$, $\alpha \in \mathbb{R}$, $v \in X$ and $\varphi \in X'$. Prove that the functional $L : M \longrightarrow \mathbb{R}$ given by

$$L[u] = \|u\|^p + \alpha\varphi(u + v) \qquad \text{for } u \in X$$

has a minimizer $u_0 \in M$.

b) Now let X be a Hilbert space and $M = X$. Determine u_0 explicitely.
Hint: Find the first variation for L. For computing the derivative $\frac{d}{d\varepsilon}\|u_0 + \varepsilon w\|^p$, use the chain rule and the definition of the norm in a Hilbert space.

To be discussed in the problem session on Tuesday, November 10, 2009

For questions concerning the problems or the problem session:

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