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WS 2009/10

4th Problem Sheet

Variational Methods and Applications to PDEs

Problem 7 (Continuation of Problem 6)

Let Ω be a bounded, open subset of $\mathbb{R}^n$ and let $f : \Omega \times \mathbb{R} \longrightarrow \mathbb{R}$ and $F : \Omega \times \mathbb{R} \longrightarrow \mathbb{R}$ be as in Problem 6. Prove that the functional $L : L^p(\Omega) \longrightarrow \mathbb{R}$ given by

$$L[u] = \int_\Omega F(x, u(x))dx, \qquad u \in L^p(\Omega).$$

is continuously Fréchet differentiable.

Problem 8

Let Ω be a bounded and open subset of $\mathbb{R}^n$ and $a \in L^2(\Omega)$. Prove that the functional $L : H_0^1(\Omega) \longrightarrow \mathbb{R}$ given by

$$L[u] = \int_\Omega \left(\frac{1}{2} |\nabla u|^2 + a(x)u \right) dx, \qquad u \in H_0^1(\Omega)$$

has a unique minimizer on $H_0^1(\Omega)$.

To be discussed in the problem session on Tuesday, November 24, 2009.