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WS 2009/10

12^{th} Problem Sheet

Variational Methods and Applications to PDEs

Problem 20

Check whether the following mappings satisfy the Palais-Smale condition:

a) $f : \mathbb{R} \longrightarrow \mathbb{R},\ f(x) = x\sin(x)$,

b) $f : \mathbb{R}^2 \longrightarrow \mathbb{R}^2,\ f(x, y) = (\cos(x) + y^2, 2xy)$,

c) $L : L^2[0, 1] \longrightarrow \mathbb{R},\ L[u] = \int_0^{1/2} u^2\,dx$.

Problem 21

Prove the following statements:

a) If $f \in C^1(\mathbb{R}^N, \mathbb{R})$ and if the mapping

$$\mathbb{R}^N \ni x \mapsto |f(x)| + |\nabla f(x)| \in \mathbb{R}$$

is coercive, then f satisfies the Palais-Smale condition.

b) If $f \in C^1(\mathbb{R}, \mathbb{R})$ is bounded from below and satisfies the Palais-Smale condition, then f is coercive and attains its infimum.

To be discussed in the problem session on Tuesday, February 2, 2010.