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13th Problem Sheet

Variational Methods and Applications to PDEs

Problem 22

Prove that the function $f : \mathbb{R}^2 \longrightarrow \mathbb{R}$ defined by

$$f(x,y) = 9(x^2+y^2) - (x^2+y^2)^2 + y\sin(x) \qquad \text{for all } (x,y) \in \mathbb{R}^2$$

has a strictly positive critical value.

Problem 23

Let $\Omega \neq \emptyset$ be an open and bounded subset of $\mathbb{R}^n$ and $2 < p < q < 2^*$ where 2^* is defined as in Problem 11. Moreover, let the functional $L : H_0^1(\Omega) \longrightarrow \mathbb{R}$ be defined by

$$L[u] = \int_\Omega \left(\frac{|\nabla u|^2}{2} - \frac{|u|^p}{p} - \frac{|u|^q}{q} \right) dx \qquad (u \in H_0^1(\Omega)).$$

a) Use the Mountain Pass Lemma to prove that L has at least one strictly positive critical value $c \in \mathbb{R}$.

b) Let $u^* \in H_0^1(\Omega)$ be a critical point of L such that $L[u^*] = c$. Find a boundary value problem that is solved by u^*.