

A numerical verification method for the existence and uniqueness of solutions of non-linear second order Sturmian boundary value problems

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Abstract

A numerical method for verifying the existence and uniqueness of solutions of second order non-linear two-point boundary value-problems with Sturmian boundary conditions

$$\begin{aligned} -u''(x) &= f(x, u(x), u'(x)), & \alpha < x < \beta, \\ \alpha_1 u(\alpha) + \alpha_2 u'(\alpha) &= 0 = \beta_1 u(\beta) + \beta_2 u'(\beta) \end{aligned}$$

in the vicinity of a given approximate numerical solution is proposed, where $\alpha, \beta, \alpha_1, \alpha_2, \beta_1, \beta_2$ are real numbers satisfying $\alpha < \beta$ and $(\alpha_1, \alpha_2) \neq (0, 0) \neq (\beta_1, \beta_2)$. The function $f(x, y, z)$ is assumed to be continuous on $[\alpha, \beta] \times \mathbb{R}^2$ and to be two times continuously differentiable with respect to y and z . All numerical computations are carried out with INTLAB.