Computational Proofs of the Stability of Lyapunov Equations

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Abstract

A prominent task in the analysis of dynamical systems in control theory is to check whether a Lyapunov equation

$$AX + XA^T = C, \;\; \text{where} A \in \mathbb{R}^{n\times n}, C \in \mathbb{R}^{n\times n} \tag{1}$$

has a positive (or negative) definite solution $X \in \mathbb{R}^{n\times n}$. The purpose of this work is to present a numerical algorithm based on interval arithmetic which, upon successful completion, proves with mathematical certainty that (1) has a symmetric positive definite solution. The algorithm then also provides correct and tight lower and upper bounds for each entry of the solution X.

The algorithm can be regarded as a variant of Krawczyk's algorithm in which we reduce the computational complexity from $\mathcal{O}(n^5)$ to $\mathcal{O}(n^3)$ together with an affine transformation that reduces the wrapping effect, and Rump's method for verifying positive-definiteness.