A computer-assisted multiplicity proof for a semilinear elliptic boundary value problem

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Abstract

In our talk we consider positive solutions of $-\Delta u - u^p = 0$ in $\Omega \subset \mathbb{R}^N$, $u = 0$ on $\partial\Omega$, $1 < p < \frac{N+2}{N-2}$ ($p > 1$ in case $N = 2$), where Ω is a domain with a hole. It has been proven that multiple solutions to this problem exist in case of Ω being an annulus $\Omega_R = \{x \in \mathbb{R}^N : R < |x| < R + 1\}$ with $R > 0$ sufficiently large. It is moreover known, that the number of (rotationally non-equivalent) positive solutions tends to infinity as $R \to \infty$. It is however unknown, for which radii bifurcation from the radial solution, which exists for all $R > 0$, occurs. Similar results, which however are all of asymptotic nature, are known for other annulus-like domains.

We consider the problem in case of the domain $\Omega_t = (-1, 1)\backslash[-t, t] \subset \mathbb{R}^2$, where $t \in (0, 1)$, $p = 3$ and ask similar questions, with the additional demand of quantification: Is there a solution similar to the radial one in the annulus case? If yes, for which size of the hole does bifurcation from this solution occur, or more generally: How many solutions exist for certain sizes of the hole?

We try to answer these questions by computer-assistance: We compute approximate solutions and use a fixed point argument to enclose true solutions nearby. To verify the assumptions needed for the fixed point theorem we make heavy use of the computer.