UNIVERSITÄT KARLSRUHE
INSTITUT FÜR ANALYSIS
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13. Exercise sheet

Functional Analysis

Deadline: Thursday, 14th Feb 2008, 15 p.m.

Exercise 49 (C)

Let X be a normed space, Y a linear subspace of X and

$$Y^{\perp} := \{\varphi \in X' : \varphi(y) = 0 \text{ for all } y \in Y\}.$$

a) Prove that the mapping defined by $T(\varphi + Y^{\perp}) := \varphi|_Y$ is an isometric isomorphism of $X'/Y^{\perp}$ onto Y'.

b) Let $q : X \to X/Y$ be the canonical mapping. Then $S(\psi) := \psi \circ q$ defines an isometric isomorphism of $(X/Y)'$ onto $Y^{\perp}$.

Exercise 50

Let X be a normed space, $V \neq \emptyset$ a closed convex subset of X and $x \in X \setminus V$. Then there is a bounded linear functional $\varphi \in X'$ such that $\operatorname{Re} \varphi(x) < \inf\{\operatorname{Re} \varphi(v) : v \in V\}$.

Exercise 51

Let (M, d) be a metric space, X a normed space and $f : M \to X$ a function. If $\varphi \circ f$ is Lipschitz continuous for all $\varphi \in X'$, then f is Lipschitz continuous.
Hint: Consider for $s, t \in M$ the linear operator $T_{s,t} : X' \to \mathbb{K}$, $T_{s,t}(\varphi) = \varphi(f(s) - f(t))$. How can you express $\|T_{s,t}\|$?

Exercise 52 (C)

Compute the adjoint operator S' of

$$S : \ell^1 \to c_0,\ (x_n)_{n\in\mathbb{N}} \mapsto \Big(\sum_{k=n}^{\infty} x_k\Big)_{n\in\mathbb{N}}.$$