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Nichtlineare Evolutionsgleichungen
2. Übungsblatt

Problem 1
Let A generate the C_0–semigroup $T(\cdot)$ on X and that $F : X \to X$ is Lipschitz on bounded subsets of X. Consider the semilinear problem

$$u'(t) = Au(t) + F(u(t)), \quad t \geq 0, \qquad u(0) = x,$$

for $x \in X$ and the maximal existence time $t^+(x) \in (0, \infty]$. Let

$$D = \{(t, x) \,|\, x \in X,\ t \in [0, t_+(x)\}.$$

Show that D is open in $\mathbb{R}_{\geq 0} \times X$. Define $\Phi : D \to X$ by $\Phi(t, x) = u(t; x)$, where $u(\,\cdot\,; x)$ is the unique mild solution of the above problem. Prove that Φ is a *local semiflow*, i.e., Ψ is continuous, we have $t^+(\Phi(s, x)) = t^+(x) - s$, and Φ satisfies

$$\Phi(0, x) = x, \qquad \Phi(t + s; x) = \Phi(t; \Phi(s; x)).$$

for all $(s, x) \in D$ and $t \in [0, t^+(x) - s)$.

Problem 2
Let A generate the C_0–semigroup $T(\cdot)$ on X. Let $F, F_n : X \to X$ for $n \in \mathbb{N}$ be Lipschitz on bounded sets and suppose that $F_n \to F$ locally uniformly as $n \to \infty$. Fix $x \in X$. Let u be the unique mild solution of

$$u'(t) = Au(t) + F(u(t)), \quad t \geq 0, \qquad u(0) = x,$$

with maximal existence time $t^+(x, F)$, and let u_n be the unique mild solution of

$$u_n'(t) = Au_n(t) + F_n(u_n(t)), \quad t \geq 0, \qquad u_n(0) = x,$$

with maximal existence time $t^+(x, F_n)$. Take any $0 < b < t^+(x, F)$. Show that $\liminf_{n\to\infty} t^+(x, F_n) > b$ and that $u_n \to u$ uniformly on $[0, b]$ as $n \to \infty$.