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3^{rd} exercise sheet
PARTIAL DIFFERENTIAL EQUATIONS
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9. Consider
$$u_t - \Delta u = 0, \quad t > 0, x \in \mathbb{R}^n$$
$$u(0, \cdot) = f \qquad \text{on } \mathbb{R}^n.$$
If $f : \mathbb{R}^n \to \mathbb{R}$ is bounded and continuos, we already know that a solution is given by
$$u(t,x) = (G_t * f)(x), \quad t > 0, x \in \mathbb{R}^n,$$
with $G_t(x) = (4\pi t)^{-n/2} e^{-|x|^2/4t}$. Show, that f is uniformly continuos if and only if
$$||u(t,\cdot) - f||_\infty \to 0 \quad \text{as } t \to 0^+.$$

10. Find a formal solution of Laplace‘s equation ($u_{xx} + u_{yy} = 0$) on the square $[0,1] \times [0,1]$ for the boundary conditions
$$\begin{aligned} u(0,y) &= 0, \\ u(1,y) &= 0, \\ u(x,0) &= 0, \\ u(x,1) &= \min\{x, 1-x\}. \end{aligned} \tag{1}$$

11. Find a formal solution of the one dimensinal wave equation ($u_{tt} = u_{xx}$) on the square $[0,\infty) \times [0,1]$ with the initial conditions
$$\begin{aligned} u(0,x) &= \sin(\pi x), \\ u_t(0,x) &= \max\{-x, x-1\}, \end{aligned} \tag{2}$$
for $x \in (0,1)$ and with the boundary conditions for $t \geq 0$
$$\begin{aligned} u(t,0) &= 0, \\ u(t,1) &= 0. \end{aligned} \tag{3}$$

12. Let $f, g : \mathbb{R} \to \mathbb{R}$ bounded with $f = g = 0$ on $(-\infty, 0)$ and
$$g|_{[0,\infty)} \in C^1[0,\infty), \qquad g, g' \in L^1(0,\infty),$$
$$f|_{[0,\infty)} \in C[0,\infty).$$
Show that $g * f \in C^1[0,\infty)$ with
$$(g * f)'(t) = g(0)f(t) + (g' * f)(t).$$
Hint: For $t > 0$ holds $g * f(t) = \int_0^t g(s)f(t-s)ds$.