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5^{th} exercise sheet
PARTIAL DIFFERENTIAL EQUATIONS
WS 2006/07

17. Complete the proof of Theorem 4.2a:
Let $G \subset \mathbb{R}^n$ open, $\varphi : G \to \mathbb{R}^n$ Lipschitz-continuous. For $\zeta \in G$ let $z(\zeta, \cdot)$ the solution of
$$z'(t) = \varphi(z(t)), \quad z(0) = \zeta.$$
Show continuity of $(\zeta, t) \mapsto z(\zeta, t)$.

18. Let $\Omega = \{(x, y) \in \mathbb{R}^2 : 0 < x < 1\}$, further let $g : (0, 1) \to \mathbb{R}$ continuos and u a solution of
$$xu_y(x, y) - yu_x(x, y) = u(x, y) \qquad \text{in } \Omega, \qquad (1)$$
$$u(x, 0) = g(x) \qquad \text{for } x \in (0, 1). \qquad (2)$$
   (i) In which region is a solution uniquely determined?
   (ii) How do we have to choose $\Gamma \subset \partial\Omega$ in order to obtain a solution u of (1) which is uniquely determined in Ω by the boundary condition $u|_\Gamma = g$, where g is a given continuous function on Γ. Is there any further assumption on g?

19. Consider the quasilinear equation
$$\begin{aligned} uu_x &= u_y && \text{in } \mathbb{R}^2, \\ u(x, 0) &= g(x) && \text{for } x \in \mathbb{R}, \end{aligned} \qquad (3)$$
for some continuos function $g : \mathbb{R} \to \mathbb{R}$. Assume, that g is not decreasing. Show that the solution does not exist for large values of y.

Hint: Show that there exist intersecting characteristics.

20. Let $g : \mathbb{R} \to \mathbb{R}$ continuous. Consider the linear equation
$$\begin{aligned} xu_x + yu_y &= u && \text{in } \mathbb{R}^2, \\ u(x, 1) &= g(x) && \text{for } x \in \mathbb{R}. \end{aligned} \qquad (4)$$
What happens for $y \to 0^+$?