Constrained Navier-Stokes Equations on 2d-Torus

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Abstract

Our research is essentially motivated by [1]. We study Navier-Stokes equations with a constraint on L^2 energy of the solution on a two dimensional domain with periodic boundary conditions. We establish the existence and uniqueness of a global solution for the following constrained Navier-Stokes equation

$$\begin{cases} \dfrac{du}{dt} - \nu\Delta u + u \cdot \nabla u = |\nabla u|^2_{L^2}\, u \\ u(0) = u_0 \end{cases} \tag{1}$$

using two different methods; Galerkin approximation and Banach Fixed Point Theorem. Here we will look at the latter method.

If time permits we will also look at the stochastic version of the above mentioned system, i.e.

$$\begin{cases} du - (\nu\Delta u + u \cdot \nabla u)\, dt = |\nabla u|^2_{L^2}\, u\, dt + \sum_{j=1}^{m} (c_j(x) \cdot \nabla u) \circ dW_j(t) \\ u(0) = u_0 \end{cases} \tag{2}$$

where $\nabla \cdot c_j = 0$ for every $j \in \{1, \ldots, m\}$ and W_j are 1D Brownian motion.

References

[1] Caglioti, E., Pulvirenti, M. and Rousset, F. On a constrained 2D Navier-Stokes Equation, 2009. Communications in Mathematical Physics, 290,651-677.