Backward stochastic Volterra integral equations in L^q spaces and its application in optimal control theory

M. Azimi, W. Grecksch
Martin Luther University Halle-Wittenberg

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Abstract

We consider a backward stochastic Volterra integral equation [BSVIE] in the Banach space $E = L^q(\mathbf{S}, \Sigma, \mu)$, where μ is σ-finte measure. The stochastic integral is defined with respect to a cylindrical Wiener process. The concept of L^p-stochastic integrability in Banach spaces will be used. We proof the existence and uniqueness of an adapted solution of BSVIE under appropriate assumptions on the coefficients of Volterra equations by using martingale representation theorem in Banach space E and Banach fixed-point theorem. Some properties of the solution are also discussed. Then for an application we consider an optimal control problem in Banach space E where the state process will be defined as forward Ito Volterra stochastic integral equation with respect to a cylindrical Wiener process. A corresponding BSVIE for the adjoint process will be derived and a duality principle between forward and backward stochastic integral equations will be calculated. Then we use maximum principle method to solve the optimal control problem.

Keywords: L^p-stochastic integrability, Unconditional martingale difference spaces, Martingale representation theorem, Co-type spaces, Cylindrical Wiener process, Stochastic maximum principle.