Kinetic maximal L^p_μ-regularity

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The *(fractional) Kolmogorov equation*

$$\begin{cases} \partial_t u + v \cdot \nabla_x u + (-\Delta_v)^{\frac{\beta}{2}} u = f, & t > 0 \\ u(0) = g, \end{cases}$$

where $u = u(t, x, v)$ and $\beta \in (0, 2]$ has gained more and more interest in the past years. This is mainly due to the following three phenomena. First, it can be seen as the prototype for many *kinetic equations*, such as the famous *Boltzmann* or the *Landau equation*. Second, even though the Laplacian only acts in half of the variables, i.e. the equation is degenerate, and despite the first-order term being unbounded, solutions of the Kolmogorov equation admit excellent regularity properties. Last but not least, it serves as a good example to study the *regularity transfer* (from v to x), a special feature of kinetic equations.
In the recent works [1, 2, 3] (joint work with Rico Zacher), we introduce the concept of *kinetic maximal L^p_μ-regularity* and prove that this property is satisfied for the (fractional) Kolmogorov equation. We show that solutions are continuous with values in the trace space and prove, in particular, that the trace space can be characterised in terms of an anisotropic Besov space. Kinetic maximal L^p_μ-regularity can be used to obtain local (in time) existence of solutions to a class of quasilinear kinetic equations by an abstract principle. We apply this principle to the *Vlasov-Poisson-Kolmogorov equation*.

References

[1] L. Niebel. Kinetic maximal $L^p_\mu(L^p)$-regularity for the fractional Kolmogorov equation with variable density, *Nonlinear Analysis*, **214** (2022).

[2] L. Niebel and R. Zacher. Kinetic maximal L^2-regularity for the (fractional) Kolmogorov equation. *Journal of Evolution Equations*, **21**, p. 3585–3612 (2021).

[3] L. Niebel and R. Zacher. Kinetic maximal L^p-regularity with temporal weights and application to quasilinear kinetic diffusion equations. *Journal of Differential Equations*, **307**, p. 29–82 (2022).