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Boundary and Eigenvalue Problems

Exercise Sheet 5

Exercise 10

Let $\Omega \subseteq \mathbb{R}^n$ be open.

(a) Show that the Sobolev spaces $W^{k,p}(\Omega)$ for $k \in \mathbb{N}$, $p \in [1, \infty]$ are Banach spaces.

(b) Deduce for $\Gamma_0 \subseteq \partial\Omega$ the spaces $W^{k,p}_{\Gamma_0}(\Omega)$ and $W^{k,p}_0(\Omega)$ are also Banach spaces.

Exercice 11

Let $\Omega \subset B_R(0)$ be open with C^1-boundary, $c \in L^\infty(\Omega)$. Give a condition on c (dependent on $R > 0$) such that the boundary value problem

$$\begin{cases} -\Delta u + x \cdot \nabla u + c(x)u = f & \text{in } \Omega \\ u(x) = 0 & \text{on } \partial\Omega \end{cases}$$

has a unique solution.

Exercise 12

Let $\Omega \subset \mathbb{R}^n$ be bounded and open with C^1-boundary and $\alpha, \beta > 0$. Consider the boundary value problem

$$\begin{cases} \Delta^2 u - \alpha\Delta u + \beta u = f & \text{in } \Omega \\ u(x) = \partial_\nu u(x) = 0 & \text{on } \partial\Omega \end{cases} \tag{1}$$

(a) Find the weak formulation to (1) in $H^2_0(\Omega)$.

(b) Show that for any right hand side $f \in L^2(\Omega)$ the problem (1) is uniquely solvable.