



Karlsruhe Institute of Technology  
Institute for Analysis

**JProf. Dr. Xian Liao**  
**Dr. Christian Zillinger**

Englerstrasse 2,  
D-76131 Karlsruhe  
E-mail: [zillinger@kit.edu](mailto:zillinger@kit.edu)

Webpage: [www.math.kit.edu/iana8/](http://www.math.kit.edu/iana8/)  
[www.uni-heidelberg.de/math/rueland/](http://www.uni-heidelberg.de/math/rueland/)

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## **Seminar: Microstructure in materials and fluid dynamics (WiSe 2020/21)**

**Lecturers: JProf. Dr. X. Liao, Dr. C. Zillinger**

**Contents:** Microstructure arises in many problems from materials science and fluid dynamics – giving rise to interesting behaviour but also to significant mathematical challenges. A famous example of this is the formation of turbulence in fluid flows. In this seminar we seek to study some aspects of the microstructure formation in prototypical settings and seek to explore how this influences the properties of the underlying PDEs.

We will begin by investigating the Euler equations describing an inviscid fluid. Due to the work of Arnold it is known that they form an infinite dimensional Hamiltonian system with infinitely many conserved quantities. This allows to formulate the equations in a very geometrical way as a variational problem on the group of volume preserving diffeomorphisms. Due to the non-convexity of this problem, it is however very difficult to attack this with the "standard" techniques of the calculus of variations. It thus becomes necessary to study relaxed versions of this problem. This will lead to very weak, measure-valued notions for solutions. We will study these "Brenier solutions" as well as "wild" convex integration solutions. In the second part of the seminar we will deal with mixing and methods of avoiding mixing in fluids and materials.

**Prerequisites:** Analysis I-III, functional analysis, introduction to PDEs.

**Date:** Mondays, 2-4 pm.

**Literature:** Daneri, Figalli: Variational models for the incompressible Euler equations.  
De Lellis, Szekelyhidi: The Euler equations as a differential inclusion.  
Kim, Koh: Two-phase solutions for one-dimensional non-convex elastodynamics.  
Villani: Hypocoercivity.

**Remarks:** If you are interested in participating in the seminar, please write an email to [zillinger@kit.edu](mailto:zillinger@kit.edu) before September 15th. There will be an online meeting to distribute topics at the end of September. The seminar will be held as an online seminar together with some groups at Heidelberg University.