1 Distance between a point and a line

Let $\mathbf{a}$ be a point and ℓ be a line $\mathbf{v}t + \mathbf{p}$, where $\mathbf{v}$ is a unit direction vector. Then

$$dist(\mathbf{a}, \ell) = dist(\mathbf{a} - \mathbf{p}, \mathbf{v}t) = |\mathbf{a} - \mathbf{p}|^2 - (\mathbf{v} \cdot \mathbf{a})^{\mathbf{2}}.$$

Another formula: If $\mathbf{p}$ and $\mathbf{q}$ are points on ℓ, then

$$dist(\mathbf{a}, \ell) = \frac{|(\mathbf{a} - \mathbf{p}) \times (\mathbf{a} - \mathbf{q})|}{|\mathbf{p} - \mathbf{q}|}.$$

2 Distance between a point and a plane

Let $\mathbf{a}$ be a point and P be a plane with normal equation $\mathbf{n} \cdot \mathbf{x} = \rho$, where $\mathbf{n}$ is a unit normal vector. Then

$$dist(\mathbf{a}, P) = |\mathbf{a} \cdot \mathbf{n} - \rho|.$$

Another formula: If $\mathbf{b} \in P$, then

$$dist(\mathbf{a}, P) = |(\mathbf{a} - \mathbf{b}) \cdot \mathbf{n}|.$$

3 Distance between two lines in $\mathbf{R}^3$

Given two line ℓ_1 and ℓ_2:

- Check whether the lines intersect by setting their parametric equations equal. If they intersect, the distance is zero.
- If they do not intersect and parallel (these can be observed by comparing the direction vectors), late any point on one line and calculate the distance to another line.
- If the lines do not intersect and are nor parallel, they belong to two parallel planes with normal vector $\mathbf{n}$. This vector is orthogonal to each of the direction vectors of the lines. Find the equation of such a plane P through ℓ_1, pick an arbitrary point $A \in \ell_2$, find the distance between A and P.

4 Distance between two planes

- Check whether the planes intersect by considering their normal vectors. If normal vectors are nor parallel or if the planes coincide, the planes intersect, the distance is zero.
- If they do not intersect, take a point in one plane and find a distance to another plane.