SUMMARY of GEOMETRIC LINEAR TRANSFORMS in $\mathbb{R}^2$.

Dilation

$$T(\mathbf{x}) = a\mathbf{x} = \begin{pmatrix} a & 0 \\ 0 & a \end{pmatrix}\mathbf{x}.$$

Reflexion with respect to x_1-axis

$$T(\mathbf{x}) = T((x_1, x_2)) = (x_1, -x_2) = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\mathbf{x}.$$

Reflexion with respect to x_2-axis

$$T(\mathbf{x}) = T((x_1, x_2)) = (-x_1, x_2) = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}\mathbf{x}.$$

Projection onto the vector $w = (w_1, w_2)$

$$T(\mathbf{x}) = proj_{\mathbf{w}}\mathbf{x} = \frac{\mathbf{w}}{|\mathbf{w}|^2}(\mathbf{x} \cdot \mathbf{w}) = \begin{pmatrix} \frac{w_1 w_1}{|\mathbf{w}|^2} & \frac{w_1 w_2}{|\mathbf{w}|^2} \\ \frac{w_2 w_1}{|\mathbf{w}|^2} & \frac{w_2 w_2}{|\mathbf{w}|^2} \end{pmatrix}\mathbf{x}.$$

Reflexion with respect to the vector $w = (w_1, w_2)$

$$T(\mathbf{x}) = 2proj_{\mathbf{w}}\mathbf{x} - \mathbf{x} = \begin{pmatrix} 2\frac{w_1 w_1}{|\mathbf{w}|^2} - 1 & 2\frac{w_1 w_2}{|\mathbf{w}|^2} \\ 2\frac{w_2 w_1}{|\mathbf{w}|^2} & 2\frac{w_2 w_2}{|\mathbf{w}|^2} - 1 \end{pmatrix}\mathbf{x}.$$

Rotation counterclockwise by the angle ϕ

$$T(\mathbf{x}) = \begin{pmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{pmatrix}\mathbf{x}.$$

Horisontal shear with coefficient k $T(\mathbf{x}) = \begin{pmatrix} 1 & k \\ 0 & 1 \end{pmatrix}\mathbf{x}.$

Building a matrix of a transform using standard basis

$$T(\mathbf{x}) = \begin{pmatrix} \\ T(\mathbf{e_1})T(\mathbf{e_2})\dots T(\mathbf{e_n}) \\ \\ \end{pmatrix}\mathbf{x}.$$