Lecture in summer semester 2016:

Geometric Numerical Integration

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http://www.math.kit.edu/ianm3/edu/geomnumint2016s/en

Scope

The numerical simulation of time-dependent processes in science and technology often leads to the problem to solve a system of ordinary differential equations (ODEs) with a suitable method. In many applications it can be shown that the exact flow exhibits certain qualitative or "geometric" properties. For example, it is well-known that the flow of a Hamiltonian system is symplectic, and that the energy remains constant along the solution although the solution itself changes in time.

When the solution or the flow is approximated by a numerical integrator, it is desirable to preserve these geometric properties (at least approximately), because reproducing the correct qualitative behavior is important in most applications. It turns out, however, that many numerical schemes destroy the structure of the solution, and that only selected methods respect the geometric properties of the dynamics. These methods are called geometric numerical integrators. In this lecture we will investigate

- why certain methods are (or are not) geometric numerical integrators,
- how to construct geometric numerical integrators,
- which properties are conserved, and in which sense,
- how structure conservation is related to the long-time error behavior of the method.

General informations

The course consists of a lecture (Wednesday and Thursday, 8:00-9:30, SR 3.61) and exercise classes (Wednesday, 8:00-9:30, computer pool -1.031, alternating with lecture).

Both the lecture and the exercise classes will be given in English. The lecture will be suited for Master students in mathematics, physics and other sciences with a basic knowledge in ordinary differential equations and Runge-Kutta methods. In particular, students should be familiar with concepts such as, e.g., order, consistency, convergence, A-stability, and so on. The course "Numerische Methoden für Differentialgleichungen" provides a good basis.

In the exercise class, students will be asked to write Matlab programs which illustrate the theoretical results presented in the lecture. The exercises can be solved in pairs or alone, at home or in class, and with the assistance of the tutor. Participants are expected to be familiar with Matlab. As KIT has a Campus License for MATLAB, all students can download and install the software.

The exams (oral, 25 mins) will take place on 24 August 2016.

References:

- Ernst Hairer, Christian Lubich, and Gerhard Wanner: *Geometric numerical integration. Structure-preserving algorithms for ordinary differential equations.*
  Second edition, Springer (Berlin, Heidelberg), 2006.

- Ernst Hairer, Christian Lubich, and Gerhard Wanner: *Geometric numerical integration illustrated by the Störmer-Verlet method.*
  Acta Numerica 12, 399-450, 2003.

- Sebastian Reich and Benedict Leimkuhler: *Simulating Hamiltonian dynamics.*
  Cambridge monographs on applied and computational mathematics 14. Cambridge University Press (Cambridge), 2004.

- Jesus Maria Sanz-Serna and Mari Paz Calvo: *Numerical Hamiltonian problems.*
  Number 7 in Applied Mathematics and Mathematical Computation. Chapman & Hall (London), 1994.