Universität Karlsruhe (TH)
Forschungsuniversität • gegründet 1825

Institut für Angewandte
und Numerische Mathematik

Winter term 2009/10

High-dimensional approximation

JProf. Dr. Tobias Jahnke

Tuesday 8:00-9:30, 1C-04, Geb. 05.20 (Allianz building)

Many important applications in financial mathematics, systems biology, chemistry, or physics require the solution of high-dimensional partial differential equations. Such problems are particularily challenging because they cannot be solved with traditional methods. The main reason is that the number of unknowns grows exponentially with the dimension such that the computational workload exceeds the capacity of most computers. For example, an equidistant discretization of the unit interval $[0,1]$ by mesh points with distance 0.1 has only 11 points $(0, 0.1, ...0.9, 1)$, but a similar discretization of the unit cube requires $11^3 = 1331$ mesh points, and a corresponding mesh on the 10-dimensional hypercube contains $11^{10} = 25,937,424,601$ mesh points. This exponential growth of the size of the problem is known as *the curse of dimesionality*.

In this lecture, we will give examples for applications which lead to high-dimensional problems and discuss properties of the corresponding equations (Master equation, Fokker-Planck equation, Schrödinger equation). Then, three strategies to avoid the curse of dimensionality will be introduced and analyzed: sparse grids, wavelet compression, and variational approximation. Special emphasis will be devoted to the question *why these approaches work* and *which assumptions have to be made*. The main goal of this lecture, however, is to convince the audience week by week that it pays to get up early to attend a lecture at 8 a.m.

The lecture will be given in English. It will be suited for students in mathematics, physics and other sciences with a basic knowledge in ordinary and partial differential equations and the corresponding numerical methods.

Literature:

- H.-J. Bungartz and M. Griebel.
  Sparse grids.
  Acta Numerica, 13:1-123, 2004.
  `http://wissrech.ins.uni-bonn.de/research/pub/griebel/sparsegrids.pdf`

- A. Cohen, W. Dahmen, and R. DeVore.
  Adaptive wavelet methods for elliptic operator equations: Convergence rates.
  Math. Comput. 70(233):27-75, 2001.
  `http://www.ams.org/mcom/2001-70-233/S0025-5718-00-01252-7/home.html`

- Christian Lubich.
  From quantum to classical molecular dynamics: Reduced models and numerical analysis.
  Zurich Lectures in Advanced Mathematics. Zürich: European Mathematical Society (EMS), 2008.

Contact: JProf. Dr. Tobias Jahnke
E-mail: tobias.jahnke@kit.edu
Office 4C-11, Geb. 05.20 (Allianz building)
Phone: 608 7982
`http://www.mathematik.uni-karlsruhe.de/ianm3/lehre/`